Motion In Two Dimension

The motion of an object is called two dimensional, if two of the three co-ordinates are required to specify the position of the object in space changes *w.r.t* time.

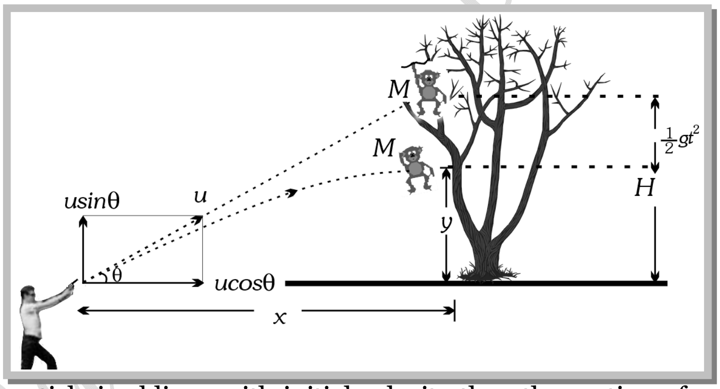
In such a motion, the object moves in a plane. For example, a billiard ball moving over the billiard table, an insect crawling over the floor of a room, earth revolving around the sun *etc.*

Two special cases of motion in two dimension are 1. Projectile motion 2. Circular motion

PROJECTILE MOTION

3.1 Introduction

A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, he will be safe but at the instant the bullet leaves the barrel of gun, if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path.



The path of motion of a as projectile motion.

If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.

3.2 Projectile

A body which is in flight through the atmosphere but is not being propelled by any fuel is called projectile.

Example: (i) A bomb released from an aeroplane in level flight

(ii) A bullet fired from a gun

(iii) An arrow released from bow

(iv) A Javelin thrown by an athlete

3.3 Assumptions of Projectile Motion

(1) There is no resistance due to air.

(2) The effect due to curvature of earth is negligible.

(3) The effect due to rotation of earth is negligible.

(4) For all points of the trajectory, the acceleration due to gravity '*g*' is constant in magnitude and direction.

3.4 Principles of Physical Independence of Motions

(1) The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts. Horizontal motion and vertical motion. These two motions take place independent of each other. This is called the principle of physical independence of motions.

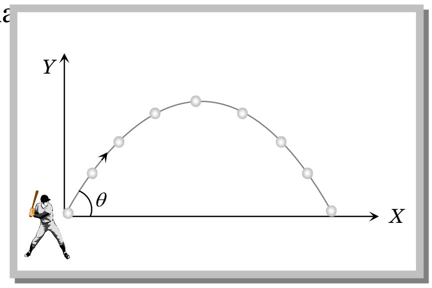
(2) The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.

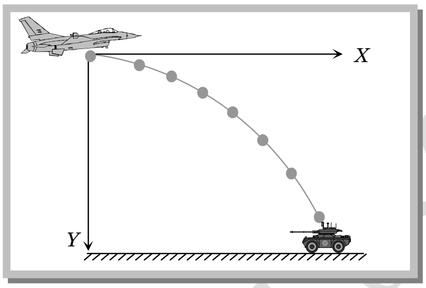
(3) The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.

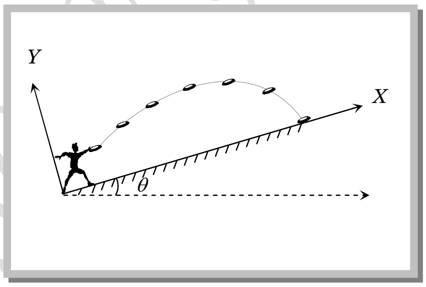
(4) The horizontal motion is a uniform motion and the vertical motion is a uniformly accelerated retarded motion.

3.5 Types of Projectile Motion

(1) Oblique projectile motion (2) Horizontal projectile motion (3) Projectile motion on an inclined pla



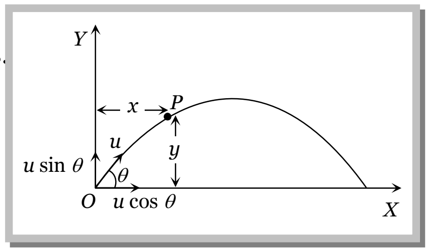




3.6 Oblique Projectile

In projectile motion, horizontal component of velocity ($u\cos\theta$), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity ($u\sin\theta$), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

(1) **Equation of trajectory :** A projectile thrown with velocity u at an angle θ with the horizontal. The velocity u can be resolved into two rectangular components.



$v\cos\theta$ component along X–axis and $u\sin\theta$ component along Y–axis.

For horizontal motion $x = u\cos\theta \times t \Rightarrow t = \dfrac{x}{u\cos\theta}$ .... (i)

For vertical motion $y = (u\sin\theta)t - \dfrac{1}{2}gt^2$ .... (ii)

From equation (i) and (ii) $y = u\sin\theta\left(\dfrac{x}{u\cos\theta}\right) - \dfrac{1}{2}g\left(\dfrac{x^2}{u^2\cos^2\theta}\right)$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

This equation shows that the trajectory of projectile is parabolic because it is similar to equation of parabola

$$y = ax - bx^2$$

Note : ❑ Equation of oblique projectile also can be written as

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$ (where R = horizontal range = $\dfrac{u^2\sin 2\theta}{g}$)

Sample problems based on trajectory

***Problem* 1.** The trajectory of a projectile is represented by $y = \sqrt{3}x - gx^2/2$. The angle of projection is

(a) 30° (b) 45° (c) 60° (d) None of these

Solution : (c) By comparing the coefficient of x in given equation with standard equation $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\tan\theta = \sqrt{3}$ $\therefore \theta = 60°$

***Problem* 2.** The path followed by a body projected along y-axis is given as by $y = \sqrt{3}x - (1/2)x^2$, if g = 10 m/s, then the initial velocity of projectile will be – (x and y are in m)

(a) $3\sqrt{10}\ m/s$ (b) $2\sqrt{10}\ m/s$ (c) $10\sqrt{3}\ m/s$ (d) $10\sqrt{2}\ m/s$

Solution : (b) By comparing the coefficient of x^2 in given equation with standard equation $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$.

$\frac{g}{2u^2\cos^2\theta} = \frac{1}{2}$

Substituting θ = 60° we get $u = 2\sqrt{10}\ m/\sec$.

***Problem* 3.** The equation of projectile is $y = 16x - \frac{5x^2}{4}$. The horizontal range is

(a) 16 m (b) 8 m (c) 3.2 m (d) 12.8 m

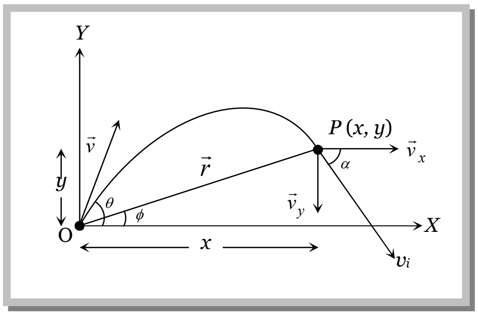
Solution : (d) Standard equation of projectile motion $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

Given equation : $y = 16x - \frac{5x^2}{4}$ or $y = 16x\left[1 - \frac{x}{64/5}\right]$

By comparing above equations $R = \frac{64}{5}$ =12.8 m.

(2) **Displacement of projectile ($\vec{r}$) :** Let the particle acquires a position P having the coordinates (x, y) just after time t from the instant of projection. The corresponding position vector of the particle at time t is $\vec{r}$ as shown in the figure.

$$\vec{r} = x\hat{i} + y\hat{j} \quad \text{....(i)}$$



The horizontal distance covered during time t is given as

$$x = v_x t \Rightarrow x = u\cos\theta\ t \quad \text{....(ii)}$$

The vertical velocity of the particle at time t is given as

$$v_y = (v_0)_y - gt, \quad \text{....(iii)}$$

Now the vertical displacement y is given as

$$y = u\sin\theta\, t - 1/2\, gt^2 \quad \text{....(iv)}$$

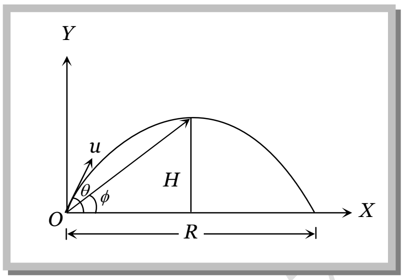
Putting the values of x and y from equation (ii) and equation (iv) in equation (i) we obtain the position vector at any time t as

$$\vec{r} = (u\cos\theta)t\,\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j} \Rightarrow r = \sqrt{(u\,t\cos\theta)^2 + \left((u\,t\sin\theta) - \frac{1}{2}gt^2\right)^2}$$

$$r = u\,t\sqrt{1 + \left(\frac{gt}{2u}\right)^2 - \frac{gt\sin\theta}{u}} \text{ and } \phi = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{ut\sin\theta - 1/2\,gt^2}{(u\,t\cos\theta)}\right) \text{ or } \phi = \tan^{-1}\left(\frac{2u\sin\theta - gt}{2u\cos\theta}\right)$$

Note : ❑ The angle of elevation ϕ of the highest point of the projectile and the angle of projection θ are related to each other as

$$\tan\phi = \frac{1}{2}\tan\theta$$

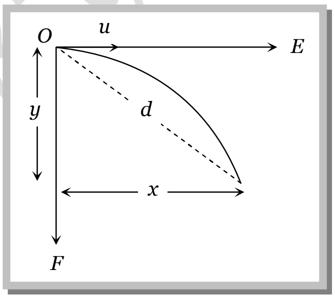


Sample problems based on displacement

***Problem* 4.** A body of mass 2 *kg* has an initial velocity of 3 *m/s* along *OE* and it is subjected to a force of 4 Newton's in *OF* direction perpendicular to *OE*. The distance of the body from *O* after 4 seconds will be

(a) 12 *m* (b) 28 *m* (c) 20 *m* (d) 48 *m*

Solution : (c) Body moves horizontally with constant initial velocity 3 *m/s* upto 4 seconds $\therefore\ x = ut = 3\times 4 = 12\,m$



and in perpendicular direction it moves under the effect of constant force with zero initial velocity upto 4 seconds.

$$\therefore\quad y = ut + \frac{1}{2}(a)t^2 = 0 + \frac{1}{2}\left(\frac{F}{m}\right)t^2 = \frac{1}{2}\left(\frac{4}{2}\right)4^2 = 16\,m$$

So its distance from *O* is given by $d = \sqrt{x^2+y^2} = \sqrt{(12)^2+(16)^2}$

$\therefore\quad d = 20\,m$

***Problem* 5.** A body starts from the origin with an acceleration of 6 m/s^2 along the *x*-axis and 8 m/s^2 along the *y*-axis. Its distance from the origin after 4 seconds will be **[MP PMT 1999]**

(a) 56 *m* (b) 64 *m* (c) 80 *m* (d) 128 *m*

Solution : (c) Displacement along *X*- axis : $x = u_x t + \frac{1}{2}a_x t^2 = \frac{1}{2}\times 6\times(4)^2 = 48\,m$

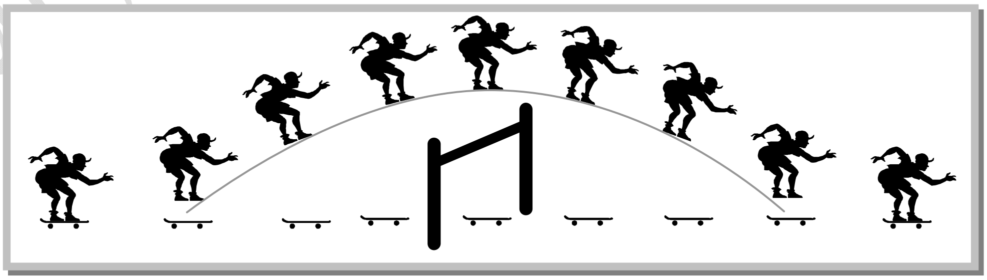
Displacement along *Y*- axis : $y = u_y t + \frac{1}{2}a_y t^2 = \frac{1}{2}\times 8\times(4)^2 = 64\,m$

Total distance from the origin $= \sqrt{x^2+y^2} = \sqrt{(48)^2+(64)^2} = 80\,m$

(3) **Instantaneous velocity *v* :** In projectile motion, vertical component of velocity changes but horizontal component of velocity remains always constant.

Example : When a man jumps over the hurdle leaving behind its skateboard then vertical component of his velocity is changing, but not the horizontal component, which matches with the skateboard velocity.

As a result, the skateboard stays underneath him, allowing him to land on it.



Let v_i be the instantaneous velocity of projectile at time *t* direction of this velocity is along the tangent to the trajectory at point *P*.

$$\vec{v}_i = v_x i + v_y\hat{j} \Rightarrow v_i = \sqrt{v_x^2+v_y^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

$$v_i = \sqrt{u^2 + g^2t^2 - 2u\,gt\sin\theta}$$

Direction of instantaneous velocity $\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$ or $\alpha = \tan^{-1}\left[\tan\theta - \frac{gt}{u}\sec\theta\right]$

(4) **Change in velocity :** Initial velocity (at projection point) $\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Final velocity (at highest point) $\vec{u}_f = u\cos\theta\,\hat{i} + 0\,\hat{j}$

(i) Change in velocity (Between projection point and highest point) $\Delta u = \vec{u}_f - \vec{u}_i = -u\sin\theta\,\hat{j}$

When body reaches the ground after completing its motion then final velocity $\vec{u}_f = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

(ii) Change in velocity (Between complete projectile motion) $\Delta u = u_f - u_i = -2u\sin\theta\,\hat{i}$

Sample problems based on velocity

Problem 6. In a projectile motion, velocity at maximum height is

(a) $\frac{u\cos\theta}{2}$ (b) $u\cos\theta$ (c) $\frac{u\sin\theta}{2}$ (d) None of these

Solution : (b) In a projectile motion at maximum height body possess only horizontal component of velocity *i.e.* $u\cos\theta$.

Problem 7. A body is thrown at angle 30° to the horizontal with the velocity of 30 *m/s*. After 1 sec, its velocity will be (in *m/s*) (g = 10 m/s^2)

(a) $10\sqrt{7}$ (b) $700\sqrt{10}$ (c) $100\sqrt{7}$ (d) $\sqrt{40}$

Solution : (a) From the formula of instantaneous velocity $v = \sqrt{u^2 + g^2t^2 - 2u\,g\,t\sin\theta}$

$$v = \sqrt{(30)^2 + (10)^2 \times 1^2 - 2\times 30\times 10\times 1\times \sin 30^o} = 10\sqrt{7}\ m/s$$

Problem 8. A projectile is fired at 30° to the horizontal. The vertical component of its velocity is 80 ms^{-1}. Its time of flight is *T*. What will be the velocity of the projectile at *t* = *T*/2

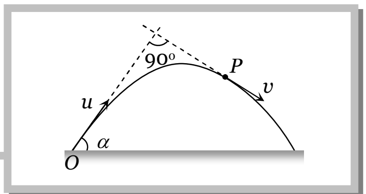
(a) 80 ms^{-1} (b) $80\sqrt{3}$ ms^{-1} (c) $(80/\sqrt{3})$ ms^{-1} (d) 40 ms^{-1}

Solution : (b) At half of the time of flight, the position of the projectile will be at the highest point of the parabola and at that position particle possess horizontal component of velocity only.

Given $u_{vertical} = u\sin\theta = 80 \Rightarrow u = \frac{80}{\sin 30^o} = 160\ m/s$

$\therefore\ u_{horizontal} = u\cos\theta = 160\ \cos 30^o = 80\sqrt{3}\ m/s.$

Problem 9. A particle is projected from point *O* with velocity *u* in a direction making an angle α with the horizontal. At any instant its position is at point *P* at right angles to the initial direction of projection. Its velocity at point *P* is

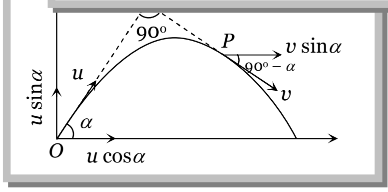


(a) $u\tan\alpha$ (b) $u\cot\alpha$ (c) $u\,\mathrm{cosec}\,\alpha$ (d) $u\sec\alpha$

Solution : (b) Horizontal velocity at point $'O' = u\cos\alpha$

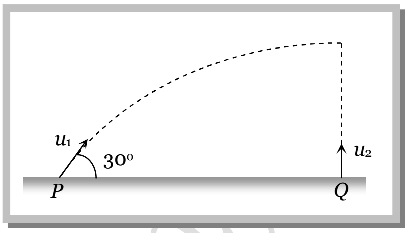
Horizontal velocity at point $'P' = v\sin\alpha$

In projectile motion horizontal component of velocity remains constant throughout the motion



$\therefore\ v\sin\alpha = u\cos\alpha \Rightarrow v = u\cot\alpha$

***Problem* 10.** A particle P is projected with velocity u_1 at an angle of 30° with the horizontal. Another particle Q is thrown vertically upwards with velocity u_2 from a point vertically below the highest point of path of P. The necessary condition for the two particles to collide at the highest point is



(a) $u_1 = u_2$

(b) $u_1 = 2u_2$

(c) $u_1 = \frac{u_2}{2}$

(d) $u_1 = 4u_2$

Solution : (b) Both particle collide at the highest point it means the vertical distance travelled by both the particle will be equal, *i.e.* the vertical component of velocity of both particle will be equal

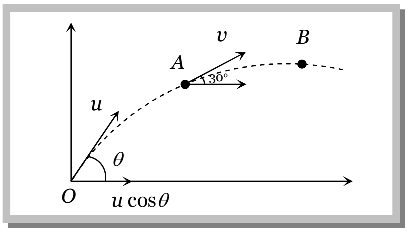
$u_1\sin 30° = u_2 \Rightarrow \frac{u_1}{2} = u_2 \quad \therefore u_1 = 2u_2$

***Problem* 11.** Two seconds after projection a projectile is travelling in a direction inclined at 30° to the horizontal after one more sec, it is travelling horizontally, the magnitude and direction of its velocity are

(a) $2\sqrt{20}\ m/\text{sec}, 60^o$ (b) $20\sqrt{3}\ m/\text{sec}, 60^o$ (c) $6\sqrt{40}\ m/\text{sec}, 30^o$ (d) $40\sqrt{6}\ m/\text{sec}, 30^o$

Solution : (b) Let in 2 *sec* body reaches upto point A and after one more *sec* upto point B.

Total time of ascent for a body is given 3 *sec i.e.* $t = \frac{u\sin\theta}{g} = 3$



$\therefore u\sin\theta = 10\times 3 = 30$ .....(i)

Horizontal component of velocity remains always constant

$u\cos\theta = v\cos 30°$ .....(ii)

For vertical upward motion between point O and A

$v\sin 30^o = u\sin\theta - g\times 2$ $[\text{Using } v = u - gt]$

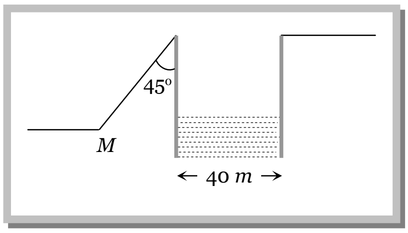
$v\sin 30^o = 30 - 20$ $[\text{As } u\sin\theta = 30]$

$\therefore v = 20\ m/s.$

Substituting this value in equation (ii) $u\cos\theta = 20\cos 30^o = 10\sqrt{3}$ .....(iii)

From equation (i) and (iii) $u = 20\sqrt{3}$ and $\theta = 60°$

***Problem* 12.** A body is projected up a smooth inclined plane (length = $20\sqrt{2}\ m$) with velocity u from the point M as shown in the figure. The angle of inclination is 45° and the top is connected to a well of diameter 40 *m*. If the body just manages to cross the well, what is the value of v



(a) $40\ ms^{-1}$

(b) $40\sqrt{2}\ ms^{-1}$

(c) $20\ ms^{-1}$

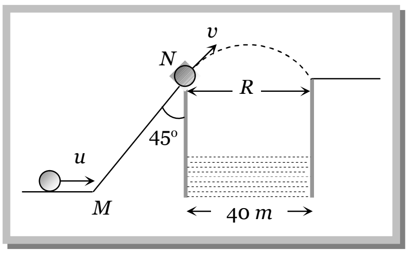
(d) $20\sqrt{2}\ ms^{-1}$

Solution : (d) At point N angle of projection of the body will be 45°. Let velocity of projection at this point is v.

If the body just manages to cross the well then Range = Diameter of well

$\frac{v^2\sin 2\theta}{g} = 40$ $[\text{As } \theta = 45°]$

$v^2 = 400 \Rightarrow v = 20\ m/s$



But we have to calculate the velocity (u) of the body at point M.

For motion along the inclined plane (from M to N)

Final velocity (v) = 20 m/s,

acceleration (a) = $-g\sin\alpha = -g\sin 45^o$, distance of inclined plane (s) = $20\sqrt{2}\ m$

$(20)^2 = u^2 - 2\frac{g}{\sqrt{2}}.20\sqrt{2}$ [Using $v^2 = u^2 + 2as$]

$u^2 = 20^2 + 400 \Rightarrow u = 20\sqrt{2}\ m/s.$

***Problem* 13.** A projectile is fired with velocity u making angle θ with the horizontal. What is the change in velocity when it is at the highest point

(a) $u\cos\theta$ (b) u (c) $u\sin\theta$ (d) $(u\cos\theta - u)$

Solution : (c) Since horizontal component of velocity remain always constant therefore only vertical component of velocity changes.

Initially vertical component $u\sin\theta$

Finally it becomes zero. So change in velocity $= u\sin\theta$

(5) **Change in momentum :** Simply by the multiplication of mass in the above expression of velocity (Article-4).

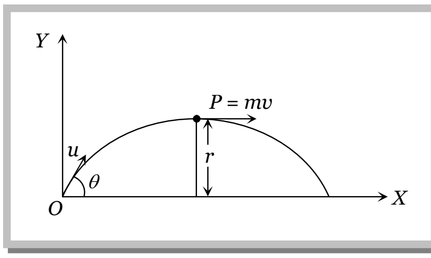
(i) Change in momentum (Between projection point and highest point) $\Delta p = \vec{p}_f - \vec{p}_i = -mu\sin\theta\,\hat{j}$

(ii) Change in momentum (For the complete projectile motion) $\Delta p = \vec{p}_f - \vec{p}_i = -2mu\sin\theta\,\hat{j}$

(6) **Angular momentum :** Angular momentum of projectile at highest point of trajectory about the point of projection is given by

$$L = mvr \qquad \left[\text{Here } r = H = \frac{u^2\sin^2\theta}{2g}\right]$$

$$\therefore \quad L = m\ u\cos\theta\frac{u^2\sin^2\theta}{2g} = \frac{m\ u^3\cos\theta\sin^2\theta}{2g}$$



Sample problems based on momentum and angular momentum

***Problem* 14.** A body of mass 0.5 kg is projected under gravity with a speed of 98 m/s at an angle of 30° with the horizontal. The change in momentum (in magnitude) of the body is

(a) 24.5 $N–s$ (b) 49.0 $N–s$ (c) 98.0 $N–s$ (d) 50.0 $N–s$

Solution : (b) Change in momentum between complete projectile motion = $2mu\sin\theta = 2\times 0.5\times 98\times\sin 30° = 49\ N–s$.

***Problem* 15.** A particle of mass 100 g is fired with a velocity 20 $m\ sec^{-1}$ making an angle of 30° with the horizontal. When it rises to the highest point of its path then the change in its momentum is

(a) $\sqrt{3}kg\ m\ sec^{-1}$ (b) 1/2 $kg\ m\ sec^{-1}$ (c) $\sqrt{2}\ kg\ m\ sec^{-1}$ (d) 1 $kg\ m\ sec^{-1}$

Solution : (d) Horizontal momentum remains always constant

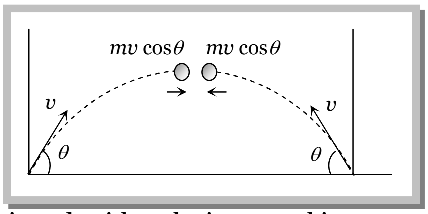
So change in vertical momentum ($\Delta\vec{p}$) = Final vertical momentum – Initial vertical momentum $= 0 - mu\sin\theta$

$|\Delta P| = 0.1\times 20\times\sin 30^o = 1\ kg\ m/sec$.

***Problem* 16.** Two equal masses (m) are projected at the same angle (θ) from two points separated by their range with equal velocities (v). The momentum at the point of their collision is

(a) Zero (b) $2\, mv \cos\theta$ (c) $-2\, mv \cos\theta$ (d) None of these

Solution : (a) Both masses will collide at the highest point of their trajectory with equal and opposite momentum. So net momentum of the system will be zero.



Problem **17.** A particle of mass m is projected with velocity v making an angle of 45^o with the horizontal. The magnitude of the angular momentum of the particle about the point of projection when the particle is at its maximum height is (where g = acceleration due to gravity)

(a) Zero (b) $mv^3/(4\sqrt{2}g)$ (c) $mv^3/(\sqrt{2}g)$ (d) $mv^2/2g$

Solution : (b) $L = \frac{m\, u^3 \cos\theta \sin^2\theta}{2g} = \frac{mv^3}{(4\sqrt{2}\, g)}$ [As $\theta = 45^o$]

Problem **18.** A body is projected from the ground with some angle to the horizontal. What happens to the angular momentum about the initial position in this motion

(a) Decreases (b) Increases

(c) Remains same (d) First increases and then decreases

Solution : (b)

Problem **19.** In case of a projectile, where is the angular momentum minimum

(a) At the starting point

(b) At the highest point

(c) On return to the ground

(d) At some location other than those mentioned above

Solution : (a)

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(7) **Time of flight :** The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.

For vertical upward motion $0 = u \sin\theta - gt \Rightarrow t = (u \sin\theta/g)$

Now as time taken to go up is equal to the time taken to come down so

Time of flight $T = 2t = \frac{2u\sin\theta}{g}$

(i) Time of flight can also be expressed as : $T = \frac{2.u_y}{g}$ (where u_y is the vertical component of initial velocity).

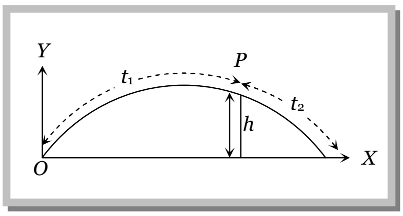
(ii) For complementary angles of projection θ and $90^o - \theta$

(a) Ratio of time of flight $= \frac{T_1}{T_2} = \frac{2u\sin\theta/g}{2u\sin(90-\theta)/g} = \tan\theta \Rightarrow \frac{T_1}{T_2} = \tan\theta$

(b) Multiplication of time of flight $= T_1 T_2 = \frac{2u\sin\theta}{g}\frac{2u\cos\theta}{g} \Rightarrow T_1 T_2 = \frac{2R}{g}$

(iii) If t_1 is the time taken by projectile to rise upto point p and t_2 is the time taken in falling from point p to ground level then $t_1 + t_2 = \frac{2u\sin\theta}{g} =$ time of flight

or $u\sin\theta = \frac{g(t_1+t_2)}{2}$



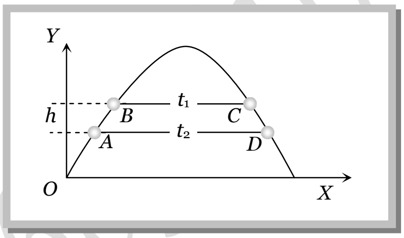
and height of the point p is given by $h = u\sin\theta\, t_1 - \frac{1}{2}gt_1^2$

$$h = g\frac{(t_1 + t_2)}{2}t_1 - \frac{1}{2}gt_1^2$$

by solving $h = \frac{g\,t_1 t_2}{2}$

(iv) If B and C are at the same level on trajectory and the time difference between these two points is t_1, similarly A and D are also at the same level and the time difference between these two positions is t_2 then

$$t_2^2 - t_1^2 = \frac{8h}{g}$$



Sample problems based on time of flight

Problem 20. For a given velocity, a projectile has the same range R for two angles of projection if t_1 and t_2 are the times of flight in the two cases then

(a) $t_1t_2 \propto R^2$ (b) $t_1t_2 \propto R$ (c) $t_1t_2 \propto \frac{1}{R}$ (d) $t_1t_2 \propto \frac{1}{R^2}$

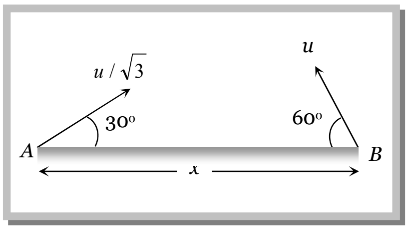
Solution : (b) As we know for complementary angles $t_1t_2 = \frac{2R}{g}$ $\therefore$ $t_1t_2 \propto R$.

Problem 21. A body is thrown with a velocity of 9.8 m/s making an angle of 30° with the horizontal. It will hit the ground after a time **[JIPMER 2001, 2002; KCET (Engg.) 2001]**

(a) 1.5 s (b) 1 s (c) 3 s (d) 2 s

Solution : (b) $T = \frac{2u\sin\theta}{g} = \frac{2\times 9.8\times \sin 30^o}{9.8} = 1sec$

Problem 22. Two particles are separated at a horizontal distance x as shown in figure. They are projected at the same time as shown in figure with different initial speed. The time after which the horizontal distance between the particles become zero is



(a) $u/2x$ (b) x/u (c) $2u/x$ (d) u/x

Solution : (b) Let x_1 and x_2 are the horizontal distances travelled by particle A and B respectively in time t.

$$x_1 = \frac{u}{\sqrt{3}}.\cos 30°\times t \quad .....(i) \qquad \text{and} \qquad x_2 = u\cos 60^o \times t \quad ......(ii)$$

$$x_1 + x_2 = \frac{u}{\sqrt{3}}.\cos 30^o \times t + u\cos 60^o \times t = ut \;\Rightarrow\; x = ut \;\therefore\; t = x/u$$

Problem 23. A particle is projected from a point O with a velocity u in a direction making an angle α upward with the horizontal. After some time at point P it is moving at right angle with its initial direction of projection. The time of flight from O to P is

(a) $\frac{u\sin\alpha}{g}$ (b) $\frac{u\,\text{cosec}\,\alpha}{g}$ (c) $\frac{u\tan\alpha}{g}$ (d) $\frac{u\sec\alpha}{g}$

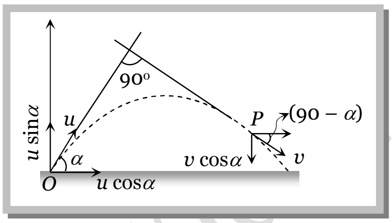
Solution : (b) When body projected with initial velocity $\vec{u}$ by making angle α with the horizontal. Then after time *t,* (at point *P*) it's direction is perpendicular to $\vec{u}$.

Magnitude of velocity at point *P* is given by $v = u\cot\alpha$. (from sample problem no. 9)

For vertical motion : Initial velocity (at point *O*) = $u\sin\alpha$

Final velocity (at point *P*) $= -v\cos\alpha = -u\cot\alpha\,\cos\alpha$

Time of flight (from point *O* to *P*) = *t*



Applying first equation of motion $v = u - gt$

$-u\cot\alpha\cos\alpha = u\sin\alpha - gt$

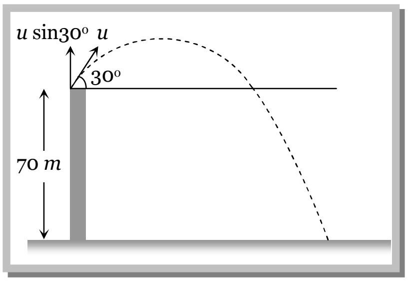
$\therefore\ t = \frac{u\sin\alpha + u\cot\alpha\cos\alpha}{g} = \frac{u}{g\sin\alpha}\left[\sin^2\alpha + \cos^2\alpha\right] = \frac{u\,\text{cosec}\,\alpha}{g}$

***Problem* 24.** A ball is projected upwards from the top of tower with a velocity 50 ms^{-1} making angle 30° with the horizontal. The height of the tower is 70 *m*. After how many seconds from the instant of throwing will the ball reach the ground

(a) 2.33 *sec* (b) 5.33 *sec* (c) 6.33 *sec* (d) 9.33 *sec*

Solution : (c) Formula for calculation of time to reach the body on the ground from the tower of height '*h*' (If it is thrown vertically up with velocity *u*) is given by

$t = \frac{u}{g}\left[1 + \sqrt{1 + \frac{2gh}{u^2}}\right]$



So we can resolve the given velocity in vertical direction and can apply the above formula.

Initial vertical component of velocity $u\sin\theta = 50\sin 30 = 25\ m/s$.

$\therefore\quad t = \frac{25}{9.8}\left[1 + \sqrt{1 + \frac{2\times 9.8\times 70}{(25)^2}}\right] = 6.33\ sec.$

***Problem* 25.** If for a given angle of projection, the horizontal range is doubled, the time of flight becomes

(a) 4 times (b) 2 times (c) $\sqrt{2}$ times (d) $1/\sqrt{2}$ times

Solution : (c) $R = \frac{u^2\sin 2\theta}{g}$ and $T = \frac{2u\sin\theta}{g}$

$\therefore\ R \propto u^2$ and $T \propto u$ (If θ and *g* are constant).

In the given condition to make range double, velocity must be increased upto $\sqrt{2}$ times that of previous value. So automatically time of flight will becomes $\sqrt{2}$ times.

***Problem* 26.** A particle is thrown with velocity *u* at an angle α from the horizontal. Another particle is thrown with the same velocity at an angle α from the vertical. The ratio of times of flight of two particles will be

(a) $\tan 2\alpha : 1$ (b) $\cot 2\alpha : 1$ (c) $\tan\alpha : 1$ (d) $\cot\alpha : 1$

Solution : (c) For first particles angle of projection from the horizontal is α. So $T_1 = \frac{2u\sin\alpha}{g}$

For second particle angle of projection from the vertical is α. it mean from the horizontal is $(90 - \alpha)$.

$\therefore T_2 = \frac{2u\sin(90-\alpha)}{g} = \frac{2u\cos\alpha}{g}$. So ratio of time of flight $\frac{T_1}{T_2} = \tan\alpha$.

***Problem* 27.** The friction of the air causes vertical retardation equal to one tenth of the acceleration due to gravity (Take *g* = 10 *ms*$^{-2}$). The time of flight will be decreased by

(a) 0% (b) 1% (c) 9% (d) 11%

Solution : (c) $T = \frac{2u\sin\theta}{g}$ $\therefore \frac{T_1}{T_2} = \frac{g_2}{g_1} = \frac{g + \frac{g}{10}}{g} = \frac{11}{10}$

Fractional decrease in time of flight $= \frac{T_1 - T_2}{T_1} = \frac{1}{11}$

Percentage decrease = 9%

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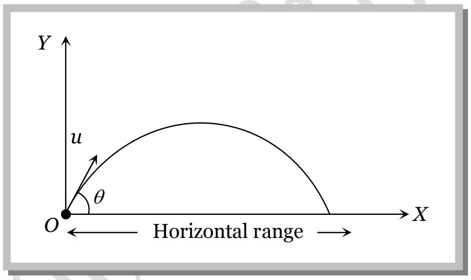
(8) **Horizontal range :** It is the horizontal distance travelled by a body during the time of flight.

So by using second equation of motion



$$R = u\cos\theta \times T = u\cos\theta \times (2u\sin\theta / g) = \frac{u^2 \sin 2\theta}{g}$$

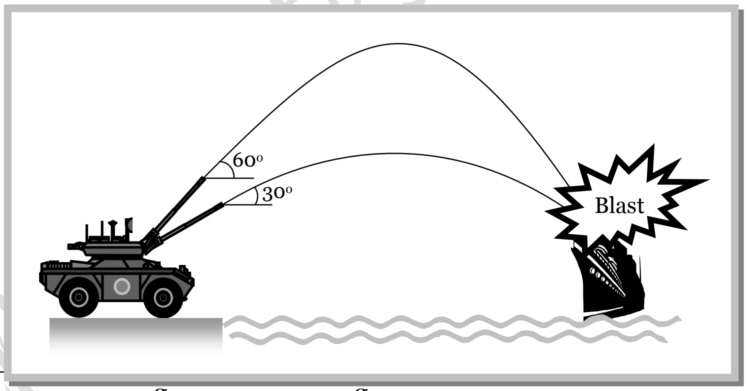
$$R = \frac{u^2 \sin 2\theta}{g}$$

(i) Range of projectile can also be expressed as :

$$R = u\cos\theta \times T = u\cos\theta \frac{2u\sin\theta}{g} = \frac{2u\cos\theta \; u\sin\theta}{g} = \frac{2u_x u_y}{g}$$

$\therefore R = \frac{2u_x u_y}{g}$ (where u_x and u_y are the horizontal and vertical component of initial velocity)

(ii) If angle of projection is changed from θ to $\theta' = (90 - \theta)$ then range remains unchanged.



$R' = \frac{u^2 \sin 2\theta}{g}$ ... g ... g

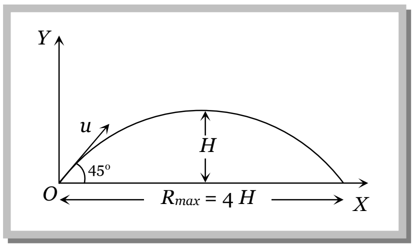
So a projectile has same range at angles of projection θ and $(90 - \theta)$, though time of flight, maximum height and trajectories are different.

These angles θ and $90^o - \theta$ are called complementary angles of projection and for complementary angles of projection ratio of range $\frac{R_1}{R_2} = \frac{u^2 \sin 2\theta / g}{u^2 \sin[2(90^o - \theta)]/g} = 1 \Rightarrow \frac{R_1}{R_2} = 1$

(iii) For angle of projection $\theta_1 = (45 - \alpha)$ and $\theta_2 = (45 + \alpha)$, range will be same and equal to $u^2 \cos 2\alpha / g$.

θ_1 and θ_2 are also the complementary angles.

(iv) Maximum range : For range to be maximum



$$\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{u^2 \sin 2\theta}{g}\right] = 0$$

$\Rightarrow \cos 2\theta = 0$ *i.e.* $2\theta = 90^o \Rightarrow \theta = 45^o$ and $R_{max} = (u^2/g)$

i.e., a projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g).

When the range is maximum, the height H reached by the projectile

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

i.e., if a person can throw a projectile to a maximum distance R_{max}, The maximum height to which it will rise is $\left(\frac{R_{max}}{4}\right)$.

(v) Relation between horizontal range and maximum height : $R = \frac{u^2 \sin 2\theta}{g}$ and $H = \frac{u^2 \sin^2\theta}{2g}$

$\therefore \quad \frac{R}{H} = \frac{u^2 \sin 2\theta / g}{u^2 \sin^2\theta / 2g} = 4\cot\theta \qquad \Rightarrow R = 4H\cot\theta$

(vi) If in case of projectile motion range R is n times the maximum height H

i.e. $\quad R = nH \quad \Rightarrow \frac{u^2 \sin 2\theta}{g} = n\frac{u^2 \sin^2\theta}{2g} \Rightarrow \tan\theta = [4/n]$ or $\theta = \tan^{-1}[4/n]$

The angle of projection is given by $\theta = \tan^{-1}[4/n]$

Note : ❑ If $R = H$ then $\theta = \tan^{-1}(4)$ or $\theta = 76^o$.

If $R = 4H$ then $\theta = \tan^{-1}(1)$ or $\theta = 45^o$.

Sample problem based on horizontal range

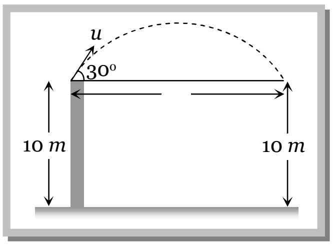
***Problem* 28.** A boy playing on the roof of a 10m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground (g = 10 m/s^2, $\sin 30^o = \frac{1}{2}$, $\cos 30^o = \frac{\sqrt{3}}{2}$) **[AIEEE 2003]**

(a) 8.66 m (b) 5.20 m (c) 4.33 m (d) 2.60 m

Solution : (a) Simply we have to calculate the range of projectile

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2\times 30°)}{10}$$

$R = 5\sqrt{3} = 8.66$ *meter*



***Problem* 29.** Which of the following sets of factors will affect the horizontal distance covered by an athlete in a long–jump event **[AMU (Engg.) 2001]**

(a) Speed before he jumps and his weight
(b) The direction in which he leaps and the initial speed
(c) The force with which he pushes the ground and his speed
(d) The direction in which he leaps and the weight

Solution : (b) Because range $= \frac{(\text{Velocity of projection})^2 \times \sin 2(\text{Angle of projection})}{g}$

***Problem* 30.** For a projectile, the ratio of maximum height reached to the square of flight time is (g = 10 ms^{-2}) **[EAMCET (Med.) 2000]**

(a) 5 : 4 (b) 5 : 2 (c) 5 : 1 (d) 10 : 1

Solution : (a) $H = \dfrac{u^2 \sin^2\theta}{2g}$ and $T = \dfrac{2u\sin\theta}{g}$ $\therefore \dfrac{H}{T^2} = \dfrac{u^2\sin^2\theta/2g}{4u^2\sin^2\theta/g^2} = \dfrac{g}{8} = \dfrac{10}{8} = \dfrac{5}{4}$

***Problem* 31.** A cricketer can throw a ball to a maximum horizontal distance of 100 *m*. The speed with which he throws the ball is (to the nearest integer)

(a) 30 ms^{-1} (b) 42 ms^{-1} (c) 32 ms^{-1} (d) 35 ms^{-1}

Solution : (c) $R_{\max} = \dfrac{u^2}{g} = 100$ (when $\theta = 45°$)

$\therefore u = \sqrt{1000} = 31.62\, m/s.$

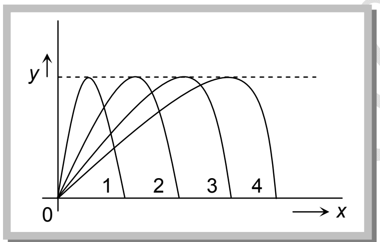
***Problem* 32.** If two bodies are projected at 30° and 60° respectively, with the same velocity, then **[CBSE PMT 2000; JIPMER 2002]**

(a) Their ranges are same (b) Their heights are same

(c) Their times of flight are same (d) All of these

Solution : (a) Because these are complementary angles.

***Problem* 33.** Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component, highest first



(a) 1, 2, 3, 4 (b) 2, 3, 4, 1 (c) 3, 4, 1, 2 (d) 4, 3, 2, 1

Solution : (d) Range ∝ horizontal component of velocity. Graph 4 shows maximum range, so football possess maximum horizontal velocity in this case.

***Problem* 34.** Four bodies *P, Q, R* and *S* are projected with equal velocities having angles of projection 15°, 30°, 45° and 60° with the horizontal respectively. The body having shortest range is **[EAMCET (Engg.) 2000]**

(a) *P* (b) *Q* (c) *R* (d) *S*

Solution : (a) Range of projectile will be minimum for that angle which is farthest from 45°.

***Problem* 35.** A particle covers 50 *m* distance when projected with an initial speed. On the same surface it will cover a distance, when projected with double the initial speed **[RPMT 2000]**

(a) 100 *m* (b) 150 *m* (c) 200 *m* (d) 250 *m*

Solution : (c) $R = \dfrac{u^2\sin 2\theta}{g}$ $\therefore R \propto u^2$ so $\dfrac{R_2}{R_1} = \left(\dfrac{u_2}{u_1}\right)^2 = \left(\dfrac{2u}{u}\right)^2 \Rightarrow R_2 = 4R_1 = 4 \times 50 = 200\, m$

***Problem* 36.** A bullet is fired from a canon with velocity 500 *m/s*. If the angle of projection is 15° and $g = 10\, m/s^2$. Then the range is **[CPMT 1997]**

(a) $25 \times 10^3\, m$ (b) $12.5 \times 10^3\, m$ (c) $50 \times 10^2\, m$ (d) $25 \times 10^2\, m$

Solution : (b) $Range\ (R) = \dfrac{u^2\sin 2\theta}{g} = \dfrac{(500)^2\sin(2\times 15)}{10} = 12500\, m = 12.5 \times 10^3\, m$

***Problem* 37.** A projectile thrown with a speed *v* at an angle *θ* has a range *R* on the surface of earth. For same *v* and *θ*, its range on the surface of moon will be

(a) *R*/6 (b) 6 *R* (c) *R*/36 (d) 36 *R*

Solution : (b) $R = \dfrac{u^2\sin 2\theta}{g}$ $\therefore R \propto 1/g$

$\frac{R_{Moon}}{R_{Earth}} = \frac{g_{Earth}}{g_{Moon}} = 6$ $\left[\because g_{Moon} = \frac{1}{6} g_{Earth}\right]$

$\therefore R_{Moon} = 6\, R_{Earth} = 6R$

***Problem* 38.** A projectile is thrown into space so as to have maximum horizontal range *R*. Taking the point of projection as origin, the co-ordinates of the point where the speed of the particle is minimum are

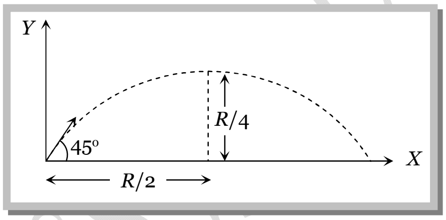
(a) (R, R) (b) $\left(R, \frac{R}{2}\right)$ (c) $\left(\frac{R}{2}, \frac{R}{4}\right)$ (d) $\left(R, \frac{R}{4}\right)$

Solution : (c) For maximum horizontal Range $\theta = 45°$

From $R = 4H\cot\theta = 4H$ [As $\theta = 45°$, for maximum range.]

Speed of the particle will be minimum at the highest point of parabola.

So the co-ordinate of the highest point will be $(R/2, R/4)$



***Problem* 39.** The speed of a projectile at the highest point becomes $\frac{1}{\sqrt{2}}$ times its initial speed. The horizontal range of the projectile will be

(a) $\frac{u^2}{g}$ (b) $\frac{u^2}{2g}$ (c) $\frac{u^2}{3g}$ (d) $\frac{u^2}{4g}$

Solution : (a) Velocity at the highest point is given by $u\cos\theta = \frac{u}{\sqrt{2}}$ (given) $\therefore \theta = 45°$

Horizontal range $R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin(2 \times 45^o)}{g} = \frac{u^2}{g}$

***Problem* 40.** A large number of bullets are fired in all directions with same speed *u*. What is the maximum area on the ground on which these bullets will spread

(a) $\pi \frac{u^2}{g}$ (b) $\pi \frac{u^4}{g^2}$ (c) $\pi^2 \frac{u^4}{g^2}$ (d) $\pi^2 \frac{u^2}{g^2}$

Solution : (b) The maximum area will be equal to area of the circle with radius equal to the maximum range of projectile

Maximum area $\pi r^2 = \pi (R_{\max})^2 = \pi \left(\frac{u^2}{g}\right)^2 = \pi \frac{u^4}{g^2}$ [As $r = R_{\max} = u^2/g$ for $\theta = 45°$]

***Problem* 41.** A projectile is projected with initial velocity $(6\hat{i} + 8\hat{j})\, m/sec$. If $g = 10\ ms^{-2}$, then horizontal range is

(a) 4.8 *metre* (b) 9.6 *metre* (c) 19.2 *metre* (d) 14.0 *metre*

Solution : (b) Initial velocity $= (6\hat{i} + 8\hat{j})\, m/s$ (given)

Magnitude of velocity of projection $u = \sqrt{u_x^2 + u_y^2} = \sqrt{6^2 + 8^2} = 10\ m/s$

Angle of projection $\tan\theta = \frac{u_y}{u_x} = \frac{8}{6} = \frac{4}{3}$ $\therefore \sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$

Now horizontal range $R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2\, 2\sin\theta\cos\theta}{g} = \frac{(10)^2 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10} = 9.6\ meter$

***Problem* 42.** A projectile thrown with an initial speed *u* and angle of projection 15° to the horizontal has a range *R*. If the same projectile is thrown at an angle of 45° to the horizontal with speed 2*u*, its range will be

(a) 12 R (b) 3 R (c) 8 R (d) 4 R

Solution : (c) $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore R \propto u^2 \sin 2\theta$

$$\frac{R_2}{R_1} = \left(\frac{u_2}{u_1}\right)^2 \left(\frac{\sin 2\theta_2}{\sin 2\theta_1}\right) \Rightarrow R_2 = R_1\left(\frac{2u}{u}\right)^2\left(\frac{\sin 90^o}{\sin 30^o}\right) = 8R_1$$

***Problem* 43.** The velocity at the maximum height of a projectile is half of its initial velocity of projection u. Its range on the horizontal plane is **[MP PET 1993]**

(a) $\sqrt{3}u^2/2g$ (b) $u^2/3g$ (c) $3u^2/2g$ (d) $3u^2/g$

Solution : (a) If the velocity of projection is u then at the highest point body posses only $u\cos\theta$

$u\cos\theta = \frac{u}{2}$ (given) $\therefore \theta = 60°$

Now $R = \frac{u^2 \sin(2\times 60°)}{g} = \frac{\sqrt{3}}{2}\frac{u^2}{g}$

***Problem* 44.** A projectile is thrown from a point in a horizontal place such that its horizontal and vertical velocity component are 9.8 m/s and 19.6 m/s respectively. Its horizontal range is

(a) 4.9 m (b) 9.8 m (c) 19.6 m (d) 39.2 m

Solution : (d) We know $R = \frac{2u_x u_y}{g} = \frac{2\times 9.8\times 19.6}{9.8} = 39.2\,m$

Where u_x = horizontal component of initial velocity, u_y = vertical component of initial velocity.

***Problem* 45.** A particle is projected with a velocity v such that its range on the horizontal plane is twice the greatest height attained by it. The range of the projectile is (where g is acceleration due to gravity)

(a) $\frac{4v^2}{5g}$ (b) $\frac{4g}{5v^2}$ (c) $\frac{v^2}{g}$ (d) $\frac{4v^2}{\sqrt{5}g}$

Solution : (a) We know $R = 4H\cot\theta$

$2H = 4H\cot\theta \Rightarrow \cot\theta = \frac{1}{2}$; $\sin\theta = \frac{2}{\sqrt{5}}$; $\cos\theta = \frac{1}{\sqrt{5}}$ [As $R = 2H$ given]

Range $= \frac{u^2 .2.\sin\theta.\cos\theta}{g} = \frac{2u^2 \frac{2}{\sqrt{5}}.\frac{1}{\sqrt{5}}}{g} = \frac{4u^2}{5g}$

***Problem* 46.** The range R of projectile is same when its maximum heights are h_1 and h_2. What is the relation between R and h_1 and h_2 **[EAMCET (Med.) 2000]**

(a) $R = \sqrt{h_1h_2}$ (b) $R = \sqrt{2h_1h_2}$ (c) $R = 2\sqrt{h_1h_2}$ (d) $R = 4\sqrt{h_1h_2}$

Solution : (d) For equal ranges body should be projected with angle θ or $(90^o - \theta)$ from the horizontal.

And for these angles : $h_1 = \frac{u^2\sin^2\theta}{2g}$ and $h_2 = \frac{u^2\cos^2\theta}{2g}$

by multiplication of both height : $h_1h_2 = \frac{u^2\sin^2\theta\cos^2\theta}{4g^2} = \frac{1}{16}\left(\frac{u^2\sin 2\theta}{g}\right)^2$

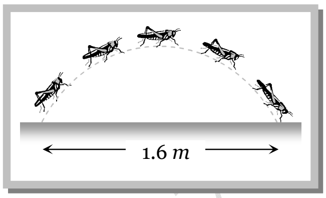
$\Rightarrow 16h_1h_2 = R^2 \Rightarrow R = 4\sqrt{h_1h_2}$

***Problem* 47.** A grasshopper can jump maximum distance 1.6 m. It spends negligible time on the ground. How far can it go in 10 *seconds*

(a) $5\sqrt{2}\,m$ (b) $10\sqrt{2}\,m$ (c) $20\sqrt{2}\,m$ (d) $40\sqrt{2}\,m$

Solution : (c) Horizontal distance travelled by grasshopper will be maximum for $\theta = 45°$

$R_{\max} = \frac{u^2}{g} = 1.6\,m \Rightarrow u = 4\,m/s.$



Horizontal component of velocity of grasshopper $u\cos\theta = 4\cos 45 = 2\sqrt{2}\,m/s$

Total distance covered by it in10 *sec.* $S = u\cos\theta \times t = 2\sqrt{2} \times 10 = 20\sqrt{2}\,m$

***Problem* 48.** A projectile is thrown with an initial velocity of $v = a\hat{i} + b\hat{j}$, if the range of projectile is double the maximum height reached by it then

(a) $a = 2b$ (b) $b = a$ (c) $b = 2a$ (d) $b = 4a$

Solution : (c) Angle of projection $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{b}{a}$ $\therefore \tan\theta = \frac{b}{a}$ ...(i)

From formula $R = 4H\cot\theta = 2H \Rightarrow \cot\theta = \frac{1}{2}$ $\therefore \tan\theta = 2$ ...(ii) [As $R = 2H$ given]

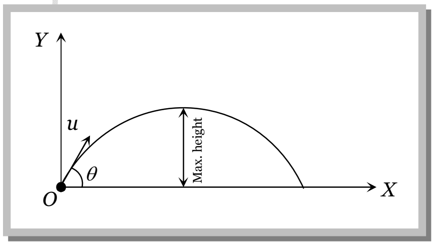
From equation (i) and (ii) $b = 2a$

(9) **Maximum height :** It is the maximum height from the point of projection, a projectile can reach.

So, by using $v^2 = u^2 + 2as$

$$0 = (u\sin\theta)^2 - 2gH$$

$$H = \frac{u^2\sin^2\theta}{2g}$$



(i) Maximum height can also be expressed as

$H = \frac{u_y^2}{2g}$ (where u_y is the vertical component of initial velocity).

(ii) $H_{\max} = \frac{u^2}{2g}$ (when $\sin^2\theta = \max = 1$ *i.e.,* $\theta = 90°$)

i.e., for maximum height body should be projected vertically upward. So it falls back to the point of projection after reaching the maximum height.

(iii) For complementary angles of projection θ and $90° - \theta$

Ratio of maximum height = $\frac{H_1}{H_2} = \frac{u^2\sin^2\theta/2g}{u^2\sin^2(90° - \theta)2g} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$

$\therefore \frac{H_1}{H_2} = \tan^2\theta$

Sample problem based on maximum height

***Problem* 49.** A cricketer can throw a ball to a maximum horizontal distance of 100 *m*. With the same effort, he throws the ball vertically upwards. The maximum height attained by the ball is **[UPSEAT 2002]**

(a) 100 *m* (b) 80 *m* (c) 60 *m* (d) 50 *m*

Solution : (d) $R_{\max} = \frac{u^2}{g} = 100\,m$ (when $\theta = 45°$)

$\therefore u^2 = 100 \times 10 = 1000$

$H_{max} = \dfrac{u^2}{2g} = \dfrac{1000}{2 \times 10} = 50\, metre.$ (when $\theta = 90°$)

Problem 50. A ball thrown by one player reaches the other in 2 *sec*. the maximum height attained by the ball above the point of projection will be about **[Pb. PMT 2002]**

(a) 10 *m* (b) 7.5 *m* (c) 5 *m* (d) 2.5 *m*

Solution : (c) $T = \dfrac{2u \sin\theta}{g} = 2\, sec$ (given) $\therefore u \sin\theta = 10$

Now $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{(10)^2}{2 \times 10} = 5\, m.$

Problem 51. Two stones are projected with the same magnitude of velocity, but making different angles with horizontal. The angle of projection of one is $\pi/3$ and its maximum height is Y, the maximum height attained by the other stone with as $\pi/6$ angle of projection is

(a) Y (b) $2Y$ (c) $3Y$ (d) $\dfrac{Y}{3}$

Solution : (d) When two stones are projected with same velocity then for complementary angles θ and $(90° - \theta)$

Ratio of maximum heights : $\dfrac{H_1}{H_2} = \tan^2\theta = \tan^2\dfrac{\pi}{3} = 3 \Rightarrow H_2 = \dfrac{H_1}{3} = \dfrac{Y}{3}$

Problem 52. If the initial velocity of a projectile be doubled. Keeping the angle of projection same, the maximum height reached by it will

(a) Remain the same (b) Be doubled (c) Be quadrupled (d) Be halved

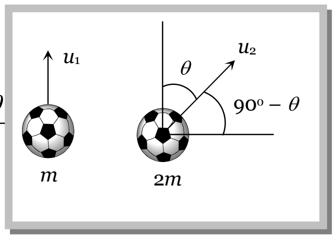
Solution : (c) $H = \dfrac{u^2 \sin 2\theta}{2g}$ $\therefore H \propto u^2$ [As θ = constant]

If initial velocity of a projectile be doubled then H will becomes 4 times.

Problem 53. Pankaj and Sudhir are playing with two different balls of masses m and $2m$ respectively. If Pankaj throws his ball vertically *up* and Sudhir at an angle θ, both of them stay in our view for the same period. The height attained by the two balls are in the ratio

(a) 2 : 1 (b) 1 : 1 (c) 1 : cosθ (d) 1 : secθ

Solution : (b) Time of flight for the ball thrown by Pankaj $T_1 = \dfrac{2u_1}{g}$



Time of flight for the ball thrown by Sudhir $T_2 = \dfrac{2u_2 \sin(90° - \theta)}{g} = \dfrac{2u_2 \cos\theta}{g}$

According to problem $T_1 = T_2 \Rightarrow \dfrac{2u_1}{g} = \dfrac{2u_2 \cos\theta}{g} \Rightarrow u_1 = u_2 \cos\theta$

Height of the ball thrown by Pankaj $H_1 = \dfrac{u_1^2}{2g}$

Height of the ball thrown by Sudhir $H_2 = \dfrac{u_2^2 \sin^2(90° - \theta)}{2g} = \dfrac{u_2^2 \cos^2\theta}{2g}$

$\therefore \dfrac{H_1}{H_2} = \dfrac{u_1^2 / 2g}{u_2^2 \cos^2\theta / 2g} = 1$ [As $u_1 = u_2 \cos\theta$]

Short Trick :
Maximum height $H \propto T^2$

$\dfrac{H_1}{H_2} = \left(\dfrac{T_1}{T_2}\right)^2$

$\therefore \dfrac{H_1}{H_2} = 1$ (As $T_1 = T_2$)

Problem 54. A boy aims a gun at a bird from a point, at a horizontal distance of 100 *m*. If the gun can impart a velocity of 500 ms^{-1} to the bullet. At what height above the bird must he aim his gun in order to hit it (take g = 10 ms^{-2})

[CPMT 1996]

(a) 20 *cm* (b) 10 *cm* (c) 50 *cm* (d) 100 *cm*

Solution : (a) Time taken by bullet to travel a horizontal distance of 100 *m* is given by $t = \frac{100}{500} = \frac{1}{5} sec$

In this time the bullet also moves downward due to gravity its vertical displacement

$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times \left(\frac{1}{5}\right)^2 = 1/5\, m$ = 20 *cm*

So bullet should be fired aiming 20 *cm* above the bird to hit it.

Problem **55.** The maximum horizontal range of a projectile is 400 *m*. The maximum height attained by it will be

(a) 100 *m* (b) 200 *m* (c) 400 *m* (d) 800 *m*

Solution : (a) $R_{max} = 400\, m$ [when $\theta = 45°$]

So from the Relation $R = 4H\cot\theta \Rightarrow 400 = 4H\cot 45° \Rightarrow H = 100\, m$.

Problem **56.** Two bodies are projected with the same velocity. If one is projected at an angle of 30° and the other at an angle of 60° to the horizontal, the ratio of the maximum heights reached is

[EAMCET (Med.) 1995; Pb. PMT 2000; AIIMS 2001]

(a) 3 : 1 (b) 1 : 3 (c) 1 : 2 (d) 2 : 1

Solution : (b) $\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{\sin^2 30^o}{\sin^2 60^o} = \frac{1}{3}$

Problem **57.** If time of flight of a projectile is 10 seconds. Range is 500 *m*. The maximum height attained by it will be

[RPMT 1997; RPET 1998]

(a) 125 *m* (b) 50 *m* (c) 100 *m* (d) 150 *m*

Solution : (a) $T = \frac{2u\sin\theta}{g} = 10 \sec \Rightarrow u\sin\theta = 50$ so $H = \frac{u^2\sin^2\theta}{2g} = \frac{(50)^2}{2\times 10} = 125\, m$.

Problem **58.** A man can throw a stone 80 *m*. The maximum height to which he can raise the stone is

(a) 10 *m* (b) 15 *m* (c) 30 *m* (d) 40 *m*

Solution : (d) The problem is different from problem no. (54). In that problem for a given angle of projection range was given and we had find maximum height for that angle.

But in this problem angle of projection can vary, $R_{max} = \frac{u^2}{g} = 80\, m$ [for $\theta = 45°$]

But height can be maximum when body projected vertically *up* $H_{max} = \frac{u^2\sin^2 90^o}{2g} = \frac{u^2}{2g} = \frac{1}{2}\left(\frac{u^2}{g}\right)$ = 40 *m*

Problem **59.** A ball is thrown at different angles with the same speed *u* and from the same points and it has same range in both the cases. If y_1 and y_2 be the heights attained in the two cases, then $y_1 + y_2 =$

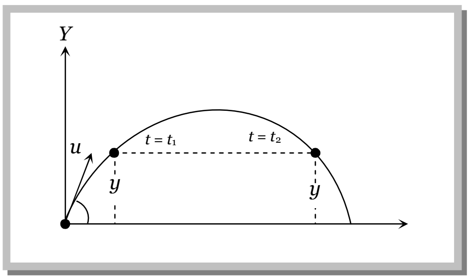
(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{g}$ (c) $\frac{u^2}{2g}$ (d) $\frac{u^2}{4g}$

Solution : (c) Same ranges can be obtained for complementary angles *i.e.* θ and 90° − θ

$y_1 = \frac{u^2\sin^2\theta}{2g}$ and $y_2 = \frac{u^2\cos^2\theta}{2g}$ $\therefore y_1 + y_2 = \frac{u^2\sin^2\theta}{2g} + \frac{u^2\cos^2\theta}{2g} = \frac{u^2}{2g}$

(10) **Projectile passing through two different points on same height at time t_1 and t_2 :** If the particle passes two points situated at equal height *y* at $t = t_1$ and $t = t_2$, then

(i) **Height (*y*):** $y = (u\sin\theta)t_1 - \frac{1}{2}gt_1^2$ .....(i)



and $y = (u\sin\theta)t_2 - \frac{1}{2}gt_2^2$ .....(ii)

Comparing equation (i) with equation (ii)

$$u\sin\theta = \frac{g(t_1 + t_2)}{2}$$

Substituting this value in equation (i)

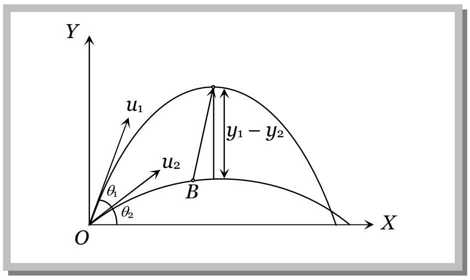
$$y = g\left(\frac{t_1 + t_2}{2}\right)t_1 - \frac{1}{2}gt_1^2 \quad \Rightarrow y = \frac{gt_1t_2}{2}$$

(ii) **Time (t_1 and t_2):** $y = u\sin\theta\, t - \frac{1}{2}gt^2$

$$t^2 - \frac{2u\sin\theta}{g}t + \frac{2y}{g} = 0 \quad \Rightarrow t = \frac{u\sin\theta}{g}\left[1 \pm \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

$$t_1 = \frac{u\sin\theta}{g}\left[1 + \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right] \text{ and } t_2 = \frac{u\sin\theta}{g}\left[1 - \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

(11) **Motion of a projectile as observed from another projectile :** Suppose two balls A and B are projected simultaneously from the origin, with initial velocities u_1 and u_2 at angle θ_1 and θ_2, respectively with the horizontal.



The instantaneous positions of the two balls are given by

Ball A : $x_1 = (u_1\cos\theta_1)t$ $\quad y_1 = (u_1\sin\theta_1)t - \frac{1}{2}gt^2$

Ball B : $x_2 = (u_2\cos\theta_2)t$ $\quad y_2 = (u_2\sin\theta_2)t - \frac{1}{2}gt^2$

The position of the ball A with respect to ball B is given by

$$x = x_1 - x_2 = (u_1\cos\theta_1 - u_2\cos\theta_2)t$$

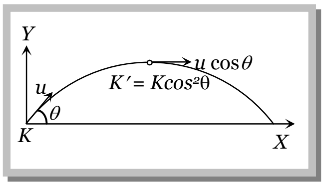
$$y = y_1 - y_2 = (u_1\sin\theta_1 - u_2\sin\theta_2)t$$

Now $\frac{y}{x} = \left(\frac{u_1\sin\theta_1 - u_2\sin\theta_2}{u_1\cos\theta_1 - u_2\cos\theta_2}\right)$ = constant

Thus motion of a projectile relative to another projectile is a straight line.

(12) **Energy of projectile :** When a projectile moves upward its kinetic energy decreases, potential energy increases but the total energy always remain constant.

If a body is projected with initial kinetic energy $K(=1/2\ mu^2)$, with angle of projection θ with the horizontal then at the highest point of trajectory



(i) **Kinetic energy** $= \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta$

$\therefore$ $\quad K' = K\cos^2\theta$

(ii) **Potential energy** $= mgH = mg\frac{u^2\sin^2\theta}{2g} = \frac{1}{2}mu^2\sin^2\theta$ $\quad \left(\text{As } H = \frac{u^2\sin^2\theta}{2g}\right)$

(iii) **Total energy** = Kinetic energy + Potential energy $= \frac{1}{2}mu^2\cos^2\theta + \frac{1}{2}mu^2\sin^2\theta$

$= \frac{1}{2}mu^2$ = Energy at the point of projection.

This is in accordance with the law of conservation of energy.

Sample problems based on energy

***Problem* 60.** A projectile is projected with a kinetic energy *K*. Its range is *R*. It will have the minimum kinetic energy, after covering a horizontal distance equal to **[UPSEAT 2002]**

(a) 0.25 *R* (b) 0.5 *R* (c) 0.75 *R* (d) *R*

Solution : (b) Projectile possess minimum kinetic energy at the highest point of the trajectory *i.e.* at a horizontal distance $R/2$.

***Problem* 61.** A projectile is fired at 30° with momentum *p*. Neglecting friction, the change in kinetic energy when it returns to the ground will be

(a) Zero (b) 30% (c) 60% (d) 100%

Solution : (a) According to law of conservation of energy, projectile acquire same kinetic energy when it comes at same level.

***Problem* 62.** A particle is projected making angle 45° with horizontal having kinetic energy *K*. The kinetic energy at highest point will be **[CBSE PMT 2000, 01; AIEEE 2002]**

(a) $\frac{K}{\sqrt{2}}$ (b) $\frac{K}{2}$ (c) $2K$ (d) K

Solution : (b) Kinetic energy at the highest point $K' = K\cos^2\theta = K\cos^2 45^o = K/2$

***Problem* 63.** Two balls of same mass are projected one vertically upwards and the other at angle 60° with the vertical. The ratio of their potential energy at the highest point is

(a) 3 : 2 (b) 2 : 1 (c) 4 : 1 (d) 4 : 3

Solution : (c) Potential energy at the highest point is given by $PE = \frac{1}{2}mu^2\sin^2\theta$

For first ball $\theta = 90°$ $\therefore$ $(PE)_1 = \frac{1}{2}mu^2$

For second ball $\theta = (90^o - 60^o) = 30°$ from the horizontal $\therefore$ $(PE)_2 = \frac{1}{2}mu^2\sin^2 30° = \frac{1}{8}mu^2$

$\therefore \quad \frac{(PE)_I}{(PE)_{II}} = 4:1$

***Problem* 64.** In the above problem, the kinetic energy at the highest point for the second ball is *K*. What is the kinetic energy for the first ball

(a) 4 *K* (b) 3 *K* (c) 2 *K* (d) Zero

Solution : (d) *KE* at the highest point $KE = \frac{1}{2}mu^2\cos^2\theta$

For first ball $\theta = 90°$ $\therefore$ $KE = 0$

***Problem* 65.** A ball is thrown at an angle θ with the horizontal. Its initial kinetic energy is 100 *J* and it becomes 30 *J* at the highest point. The angle of projection is

(a) 45° (b) 30° (c) $\cos^{-1}(3/10)$ (d) $\cos^{-1}(\sqrt{3/10})$

Solution : (d) *KE* at highest point $K' = K\cos^2\theta$

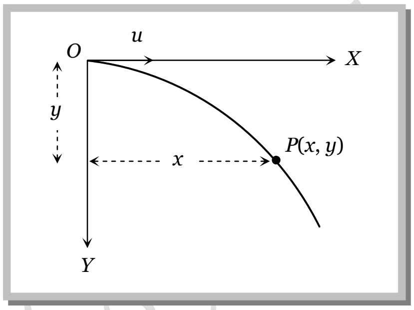
$$30 = 100\cos^2\theta \Rightarrow \cos^2\theta = \frac{3}{10} \Rightarrow \theta = \cos^{-1}\left(\sqrt{\frac{3}{10}}\right)$$

3.7 Horizontal Projectile

A body be projected horizontally from a certain height 'y' vertically above the ground with initial velocity u. If friction is considered to be absent, then there is no other horizontal force which can affect the horizontal motion. The horizontal velocity therefore remains constant and so the object covers equal distance in horizontal direction in equal intervals of time.

(1) **Trajectory of horizontal projectile :** The horizontal displacement x is governed by the equation

$$x = ut \Rightarrow t = \frac{x}{u} \quad \text{.... (i)}$$



The vertical displacement y is governed by $y = \frac{1}{2}gt^2$ .... (ii)

(since initial vertical velocity is zero)

By substituting the value of t in equation (ii) $y = \frac{1}{2}\frac{gx^2}{u^2}$

Sample problems based on trajectory

***Problem* 66.** An aeroplane is flying at a constant horizontal velocity of 600 *km/hr* at an elevation of 6 *km* towards a point directly above the target on the earth's surface. At an appropriate time, the pilot releases a ball so that it strikes the target at the earth. The ball will appear to be falling

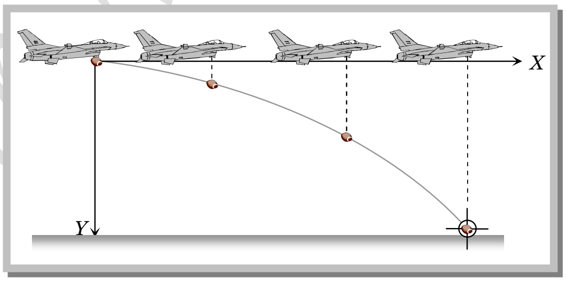
(a) On a parabolic path as seen by pilot in the plane

(b) Vertically along a straight path as seen by an observer on the ground near the target

(c) On a parabolic path as seen by an observer on the ground near the target

(d) On a zig-zag path as seen by pilot in the plane

Solution : (c)



The path of the ball appears parabolic to a observer near the target because it is at rest. But to a Pilot the path appears straight line because the horizontal velocity of aeroplane and the ball are equal, so the relative horizontal displacement is zero.

***Problem* 67.** The barrel of a gun and the target are at the same height. As soon as the gun is fired, the target is also released. In which of the following cases, the bullet will not strike the target

(a) Range of projectile is less than the initial distance between the gun and the target

(b) Range of projectile is more than the initial distance between the gun and the target

(c) Range of projectile is equal to the initial distance between the gun and target

(d) Bullet will always strike the target

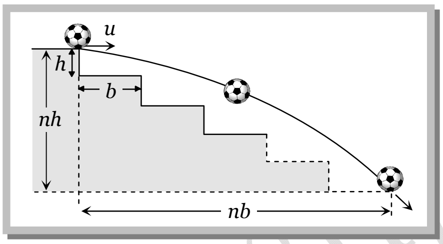
Solution : (a) Condition for hitting of bullet with target initial distance between the gun and target $\leq$ Range of projectile.

***Problem* 68.** A ball rolls off top of a staircase with a horizontal velocity u m/s. If the steps are h *metre* high and b *mere* wide, the ball will just hit the edge of *nth* step if n equals to

(a) $\frac{hu^2}{gb^2}$ (b) $\frac{u^2 8}{gb^2}$ (c) $\frac{2hu^2}{gb^2}$ (d) $\frac{2u^2 g}{hb^2}$

Solution : (c) By using equation of trajectory $y = \frac{gx^2}{2u^2}$ for given condition

$$nh = \frac{g(nb)^2}{2u^2} \quad \therefore n = \frac{2hu^2}{gb^2}$$

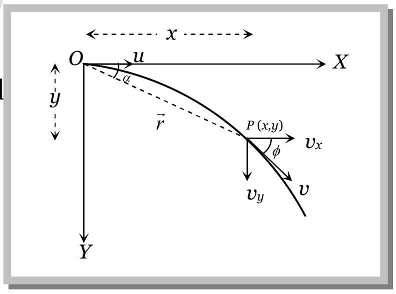


(2) **Displacement of Projectile ($\vec{r}$)** : After time *t*, horizontal displacement $x = ut$ and vertical displacement $y = \frac{1}{2}gt^2$.

So, the position vector $\vec{r} = ut\,\hat{i} - \frac{1}{2}gt^2\,\hat{j}$

Therefore $r = ut\sqrt{1 + \left(\frac{gt}{2u}\right)^2}$ and $\alpha = \tan^{-1}\left(\frac{gt}{2u}\right)$

$\alpha = \tan^{-1}\left(\sqrt{\frac{gy}{2}}\Big/u\right)$ $\left(\text{as } t = \sqrt{\frac{2y}{g}}\right)$



(3) **Instantaneous velocity :** Throughout the motion, the horizontal component of the velocity is $v_x = u$.

The vertical component of velocity increases with time and is given by

$v_y = 0 + g\,t = g\,t$ (From $v = u + g\,t$)

So, $\vec{v} = v_x\hat{i} - v_y\hat{j} = \vec{v} = u\,\hat{i} - g\,t\,\hat{j}$

i.e. $v = \sqrt{u^2 + (gt)^2} = u\sqrt{1 + \left(\frac{gt}{u}\right)^2}$

Again $\vec{v} = u\hat{i} - \sqrt{2gy}\,\hat{j}$

i.e. $v = \sqrt{u^2 + 2gy}$

Direction of instantaneous velocity : $\tan\phi = \frac{v_y}{v_x} \Rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{\sqrt{2gy}}{u}\right)$ or $\phi = \tan^{-1}\left(\frac{gt}{u}\right)$

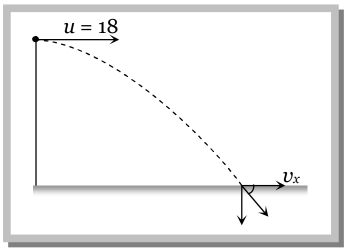
Where ϕ is the angle of instantaneous velocity from the horizontal.

Sample problems based on velocity

***Problem* 69.** A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at angle 45^o. What is the vertical component of velocity when it strikes the ground

(a) 9 ms^{-1} (b) $9\sqrt{2}$ ms^{-1} (c) 18 ms^{-1} (d) 18 $\sqrt{2}$ ms^{-1}

Solution : (c) When the body strikes the ground



$$\tan 45^o = \frac{v_y}{v_x} = \frac{v_y}{18} = 1$$

$$v_y = 18\ m/s.$$

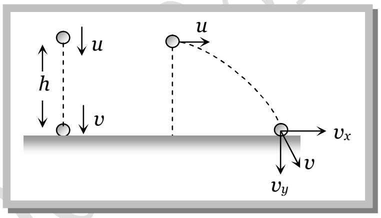
***Problem* 70.** A man standing on the roof of a house of height h throws one particle vertically downwards and another particle horizontally with the same velocity u. The ratio of their velocities when they reach the earth's surface will be

(a) $\sqrt{2gh+u^2} : u$ (b) 1 : 2 (c) 1 : 1 (d) $\sqrt{2gh+u^2} : \sqrt{2gh}$

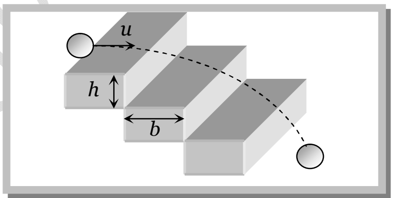
Solution : (c) For first particle : $v^2 = u^2 + 2gh \Rightarrow v = \sqrt{u^2 + 2gh}$

For second particle : $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \left(\sqrt{2gh}\right)^2} = \sqrt{u^2 + 2gh}$

So the ratio of velocities will be 1 : 1.



***Problem* 71.** A staircase contains three steps each 10 *cm* high and 20 *cm* wide. What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the lowest plane



(a) 0.5 *m/s* (b) 1 *m/s* (c) 2 *m/s* (d) 4 *m/s*

Solution : (c) Formula for this condition is given by $n = \frac{2hu^2}{gb^2}$

where h = height of each step, b = width of step, u = horizontal velocity of projection, n = number of step.

$$\Rightarrow 3 = \frac{2 \times 10 \times u^2}{10 \times 20^2} \Rightarrow u^2 = 200\ cm/sec = 2\ m/sec$$

(4) **Time of flight :** If a body is projected horizontally from a height h with velocity u and time taken by the body to reach the ground is T, then

$$h = 0 + \frac{1}{2} gT^2 \text{ (for vertical motion)}$$

$$T = \sqrt{\frac{2h}{g}}$$

Sample problems based on time of flight

***Problem* 72.** Two bullets are fired simultaneously, horizontally and with different speeds from the same place. Which bullet will hit the ground first

(a) The faster one (b) Depends on their mass

(c) The slower one (d) Both will reach simultaneously

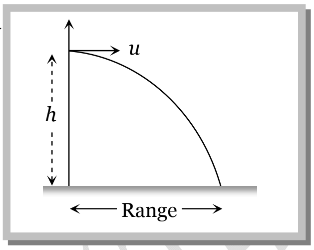
Solution : (d)

***Problem* 73.** An aeroplane is flying at a height of 1960 *m* in horizontal direction with a velocity of 360 *km/hr*. When it is vertically above the point. *A* on the ground, it drops a bomb. The bomb strikes a point *B* on the ground, then the time taken by the bomb to reach the ground is

(a) $20\sqrt{2}$ *sec* (b) 20 *sec* (c) $10\sqrt{2}$ *sec* (d) 10 *sec*

Solution : (b) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20\ sec$

(5) **Horizontal range :** Let R is the horizontal distance travelled by the body



$$R = uT + \frac{1}{2}\ 0\ T^2 \text{ (for horizontal motion)}$$

$$R = u\sqrt{\frac{2h}{g}}$$

Sample problems based on horizontal range

***Problem* 74.** A bomb is dropped on an enemy post by an aeroplane flying with a horizontal velocity of 60 *km/hr* and at a height of 490 *m*. How far the aeroplane must be from the enemy post at time of dropping the bomb, so that it may directly hit the target. ($g = 9.8\ m/s^2$)

(a) $\frac{100}{3} m$ (b) $\frac{500}{3} m$ (c) $\frac{200}{3} m$ (d) $\frac{400}{3} m$

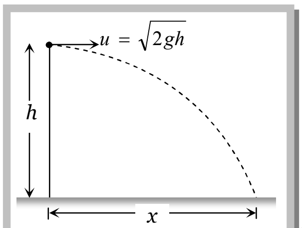
Solution : (b) $S = u \times t = u \times \sqrt{\frac{2h}{g}} = 60 \times \frac{5}{18} \times \sqrt{\frac{2 \times 490}{9.8}} = \frac{500}{3} m$

***Problem* 75.** A body is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of tower at a distance x from the tower. The value of x is

(a) h (b) $\frac{h}{2}$ (c) $2h$ (d) $\frac{2h}{3}$

Solution : (c) $x = u \times \sqrt{\frac{2h}{g}} = \sqrt{2gh} \times \sqrt{\frac{2h}{g}}$

$\therefore\ x = 2h$



***Problem* 76.** An aeroplane moving horizontally with a speed of 720 *km/h* drops a fo t a height of 396.9 *m*. The time taken by a food packet to reach the ground and its horizontal range is (Take $g = 9.8\ m/sec^2$)

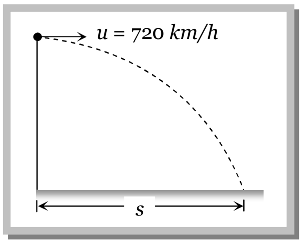
(a) 3 *sec* and 2000 *m* (b) 5 *sec* and 500 *m* (c) 8 *sec* and 1500 *m* (d) 9 *sec* and 1800 *m*

Solution : (d) Time of descent $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 396.9}{9.8}}$

$\Rightarrow t = 9\ sec$

and horizontal distance $S = u \times t$

$S = \left(\frac{720 \times 5}{18}\right) \times 9 = 1800\ m$

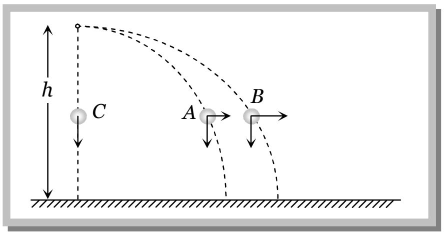


(6) If projectiles A and B are projected horizontally with different initial velocity from same height and third particle C is dropped from same point then

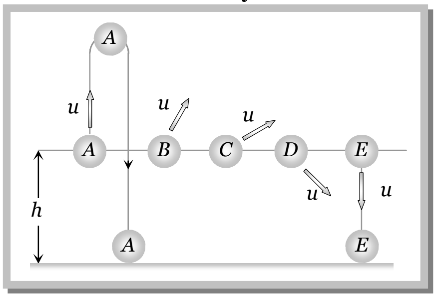
(i) All three particles will take equal time to reach the ground.

(ii) Their net velocity would be different but all three particle possess same vertical component of velocity.

(iii) The trajectory of projectiles A and B will be straight line *w.r.t.* particle C.



(7) If various particles thrown with same initial velocity but indifferent direction then



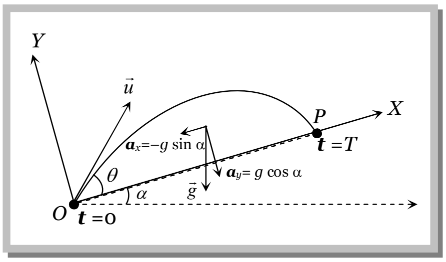
(i) They strike the ground with same speed at different times irrespective of their initial direction of velocities.

(ii) Time would be least for particle E which was thrown vertically downward.

(iii) Time would be maximum for particle A which was thrown vertically upward.

3.8 Projectile Motion on an Inclined Plane

Let a particle be projected up with a speed u from an inclined plane which makes an angle α with the horizontal velocity of projection makes an angle θ with the inclined plane.



We have taken reference x-axis in the direction of plane.

Hence the component of initial velocity parallel and perpendicular to the plane are equal to $u\cos\theta$ and $u\sin\theta$ respectively *i.e.*

$u_{\parallel} = u\cos\theta$ and $u_{\perp} = u\sin\theta$.

The component of g along the plane is $g\sin\alpha$ and perpendicular to the plane is $g\cos\alpha$ as shown in the figure *i.e.* $a_{\parallel} = -g\sin\alpha$ and $a_{\perp} = g\cos\alpha$.

Therefore the particle decelerates at a rate of $g\sin\alpha$ as it moves from O to P.

(1) **Time of flight :** We know for oblique projectile motion $T = \dfrac{2u\sin\theta}{g}$

or we can say $T = \dfrac{2u_{\perp}}{a_{\perp}}$

$\therefore$ Time of flight on an inclined plane $T = \dfrac{2u\sin\theta}{g\cos\alpha}$

(2) **Maximum height :** We know for oblique projectile motion $H = \dfrac{u^2\sin^2\theta}{2g}$

or we can say $H = \dfrac{u_{\perp}^2}{2a_{\perp}}$

$\therefore$ Maximum height on an inclined plane $H = \dfrac{u^2\sin^2\theta}{2g\cos\alpha}$

(3) **Horizontal range :** For one dimensional motion $s = ut + \dfrac{1}{2}at^2$

Horizontal range on an inclined plane $R = u_{\parallel}\ T + \frac{1}{2} a_{\parallel}\ T^2$

$$R = u\cos\theta\, T - \frac{1}{2} g\sin\alpha\ T^2$$

$$R = u\cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2} g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

By solving $R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$

(i) Maximum range occurs when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$

(ii) The maximum range along the inclined plane when the projectile is thrown upwards is given by

$$R_{\max} = \frac{u^2}{g(1+\sin\alpha)}$$

(iii) The maximum range along the inclined plane when the projectile is thrown downwards is given by

$$R_{\max} = \frac{u^2}{g(1-\sin\alpha)}$$

Sample problem based on inclined projectile

***Problem* 77.** For a given velocity of projection from a point on the inclined plane, the maximum range down the plane is three times the maximum range up the incline. Then, the angle of inclination of the inclined plane is

(a) 30^o (b) 45^o (c) 60^o (d) 90^o

Solution : (a) Maximum range up the inclined plane $(R_{\max})_{up} = \frac{u^2}{g(1+\sin\alpha)}$

Maximum range down the inclined plane $(R_{\max})_{down} = \frac{u^2}{g(1-\sin\alpha)}$

and according to problem : $\frac{u^2}{g(1-\sin\alpha)} = 3\times\frac{u^2}{g(1+\sin\alpha)}$

By solving $\alpha = 30^o$

***Problem* 78.** A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30^o$, and the angle of the barrel to the horizontal $\beta = 60^o$. The initial velocity v of the shell is 21 *m/sec*. Then distance of point from the gun at which shell will fall

(a) 10 *m* (b) 20 *m* (c) 30 *m* (d) 40 *m*

Solution : (c) Here $u = 21\ m/sec$, $\alpha = 30^o$, $\theta = \beta - \alpha = 60^o - 30^o = 30^o$

Maximum range $R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha} = \frac{2\times(21)^2\times\sin 30^o\cos 60^o}{9.8\times\cos^2 30^o} = 30\ m$

***Problem* 79.** The maximum range of rifle bullet on the horizontal ground is 6 *km* its maximum range on an inclined of 30^o will be

(a) 1 *km* (b) 2 *km* (c) 4 *km* (d) 6 *km*

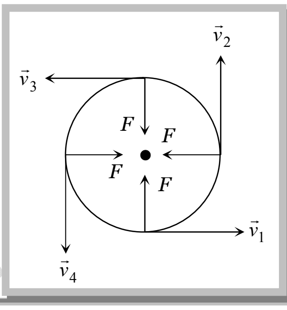
Solution : (c) Maximum range on horizontal plane $R = \frac{u^2}{g} = 6km$ (given)

Maximum range on a inclined plane $R_{\max} = \frac{u^2}{g(1+\sin\alpha)}$

Putting $\alpha = 30^o$ $R_{\max} = \frac{u^2}{g(1+\sin 30^o)} = \frac{2}{3}\left(\frac{u^2}{g}\right) = \frac{2}{3}\times 6 = 4\ km$.

CIRCULAR MOTION

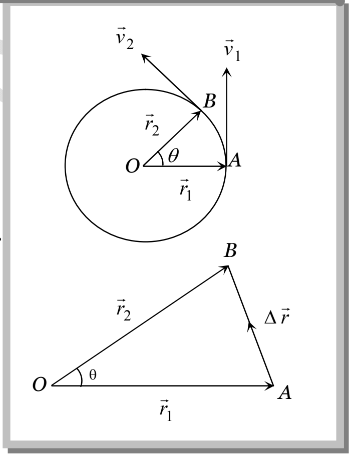
Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.



Since this force is always at right angles to the displacement due to the initial velocity therefore no work is done by the force on the particle. Hence, its kinetic energy and thus speed is unaffected. But due to simultaneous action of the force and the velocity the particle follows resultant path, which in this case is a circle. Circular motion can be classified into two types – Uniform circular motion and non-uniform circular motion.

3.9 Variables of Circular Motion

(1) **Displacement and distance :** When particle moves in a circular path describing an angle θ during time t (as shown in the figure) from the position A to the position B, we see that the magnitude of the position vector $\vec{r}$ (that is equal to the radius of the circle) remains constant. *i.e.,* $|\vec{r}_1| = |\vec{r}_2| = r$ and the direction of the position vector changes from time to time.



(i) Displacement **:** The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position A to the position B is given by referring the figure.

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \Delta r = |\Delta\vec{r}| = |\vec{r}_2 - \vec{r}_1| \quad \Delta r = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta}$

Putting $r_1 = r_2 = r$ we obtain

$$\Delta r = \sqrt{r^2 + r^2 - 2r.r\cos\theta}$$

$$\Rightarrow \Delta r = \sqrt{2r^2(1-\cos\theta)} = \sqrt{2r^2\left(2\sin^2\frac{\theta}{2}\right)}$$

$$\Delta r = 2r\sin\frac{\theta}{2}$$

(ii) Distance **:** The distanced covered by the particle during the time t is given as

d = length of the arc $AB = r\,\theta$

(iii) Ratio of distance and displacement **:** $\dfrac{d}{\Delta r} = \dfrac{r\theta}{2r\sin\theta/2} = \dfrac{\theta}{2}\text{cosec}\,(\theta/2)$

Sample problems based on distance and displacement

***Problem* 80.** A particle is rotating in a circle of radius r. The distance traversed by it in completing half circle would be

(a) r (b) πr (c) $2\pi r$ (d) Zero

Solution : (b) Distance travelled by particle = Semi-circumference = πr.

***Problem* 81.** An athlete completes one round of a circular track of radius 10 *m* in 40 *sec*. The distance covered by him in 2 min 20 sec is **[Kerala PMT 2002]**

(a) 70 *m* (b) 140 *m* (c) 110 *m* (d) 220 *m*

Solution : (d) No. of revolution $(n) = \dfrac{\text{Total time of motion}}{\text{Time period}} = \dfrac{140\text{ sec}}{40\text{ sec}} = 3.5$

Distance covered by an athlete in revolution $= n(2\pi r) = 3.5(2\pi r) = 3.5 \times 2 \times \dfrac{22}{7} \times 10 = 220\,m$.

***Problem* 82.** A wheel covers a distance of 9.5 *km* in 2000 revolutions. The diameter of the wheel is **[RPMT 1999; BHU 2000]**

(a) 15 *m* (b) 7.5 *m* (c) 1.5 *m* (d) 7.5 *m*

Solution : (c) Distance $= n(2\pi r) \Rightarrow 9.5\times10^3 = 2000\times(\pi D) \Rightarrow D = \frac{9.5\times10^3}{2000\times\pi} = 1.5\, m.$

(2) **Angular displacement (θ) :** The angle turned by a body moving on a circle from some reference line is called angular displacement.

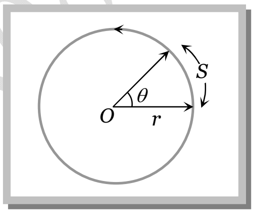
(i) Dimension = $[M^0L^0T^0]$ (as $\theta = arc/\text{radius}$).

(ii) Units = Radian or Degree. It is some times also specified in terms of fraction or multiple of revolution.

(iii) 2π rad $= 360^o = 1$ Revolution

(iv) Angular displacement is a axial vector quantity.

Its direction depends upon the sense of rotation of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.



(v) Relation between linear displacement and angular displacement $\vec{s} = \vec{\theta}\times\vec{r}$

or $s = r\theta$

Sample problem based on angular displacement

***Problem* 83.** A flywheel rotates at constant speed of 3000 *rpm*. The angle described by the shaft in radian in one second is

(a) 2π (b) 30π (c) 100π (d) 3000π

Solution : (c) Angular speed $= 3000\, rpm = 50\, rps = 50\times2\pi\, rad/\sec = 100\pi\, rad/\sec$

i.e. angle described by the shaft in one second is 100π *rad*.

***Problem* 84.** A particle completes 1.5 revolutions in a circular path of radius 2 *cm*. The angular displacement of the particle will be – (in radian)

(a) 6π (b) 3π (c) 2π (d) π

Solution : (b) 1 revolution mean the angular displacement of $2\pi\, rad$

$\therefore$ 1.5 revolution means $1.5\times2\pi = 3\pi\, rad$.

(3) **Angular velocity (ω) :** Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

(i) Angular velocity $\omega = \frac{\text{angle traced}}{\text{time taken}} = \underset{\Delta t\to0}{Lt}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

$$\therefore \quad \omega = \frac{d\theta}{dt}$$

(ii) Dimension : $[M^0L^0T^{-1}]$

(iii) Units : Radians per second ($rad.s^{-1}$) or Degree per second.

(iv) Angular velocity is an axial vector.

(v) Relation between angular velocity and linear velocity $\vec{v} = \vec{\omega}\times\vec{r}$

Its direction is the same as that of $\Delta\theta$. For anticlockwise rotation of the point object on the circular path, the direction of ω, according to Right hand rule is along the axis of circular path directed upwards. For clockwise rotation of the point object on the circular path, the direction of ω is along the axis of circular path directed downwards.

Note : ❑ It is important to note that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place in a plane perpendicular to it.

(vi) For uniform circular motion ω remains constant where as for non-uniform motion ω varies with respect to time.

Sample problems based on angular velocity

***Problem* 85.** A scooter is going round a circular road of radius 100 *m* at a speed of 10 *m/s*. The angular speed of the scooter will be

(a) 0.01 *rad/s* (b) 0.1 *rad/s* (c) 1 *rad/s* (d) 10 *rad/s*

Solution : (b) $\omega = \frac{v}{r} = \frac{10}{100} = 0.1\, rad/sec$

***Problem* 86.** The ratio of angular velocity of rotation of minute hand of a clock with the angular velocity of rotation of the earth about its own axis is

(a) 12 (b) 6 (c) 24 (d) None of these

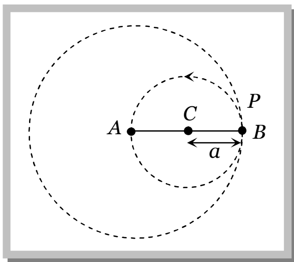
Solution : (c) $\omega_{Minute\ hand} = \frac{2\pi}{60}\frac{rad}{\min}$ and $\omega_{Earth} = \frac{2\pi}{24}\frac{Rad}{hr} = \frac{2\pi}{24\times 60}\frac{rad}{\min}$ $\therefore \frac{\omega_{Minute\ hand}}{\omega_{Earth}} = 24:1$

***Problem* 87.** A particle *P* is moving in a circle of radius '*a*' with a uniform speed *v*. *C* is the centre of the circle and *AB* is a diameter. When passing through *B* the angular velocity of *P* about *A* and *C* are in the ratio **[NCERT 1982]**

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

Solution : (b) Angular velocity of *P* about *A*

$\omega_A = \frac{v}{2a}$ Angular velocity of *P* about *C* $\omega_C = \frac{v}{a}$ $\therefore \frac{\omega_A}{\omega_C} = 1:2$



***Problem* 88.** The length of the seconds hand of a watch is 10 *mm*. What is the change in [illegible] atch after 15 *seconds*

(a) Zero (b) $(10\pi/2)\ mms^{-1}$ (c) $(20/\pi)\ mms^{-1}$ (d) $10\sqrt{2}\ mms^{-1}$

Solution : (a) Angular speed of seconds hand of watch is constant and equal to $\frac{2\pi}{60}\frac{rad}{sec} = \frac{\pi}{30} rad/sec$. So change in angular speed will be zero.

(4) **Change in velocity :** We want to know the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion as it moves from *A* to *B* during time *t* as shown in figure. The change in velocity vector is given as

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

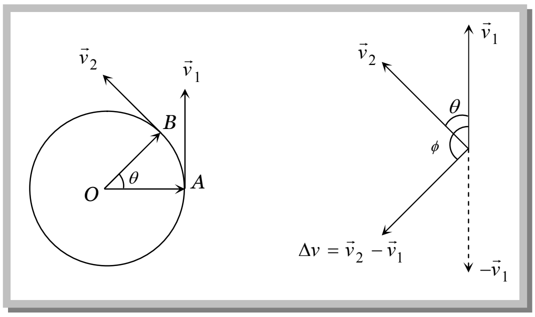
or $|\Delta\vec{v}| = |\vec{v}_2 - \vec{v}_1| \Rightarrow \Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$

For uniform circular motion $v_1 = v_2 = v$

So $\Delta v = \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$

The direction of $\Delta\vec{v}$ is shown in figure that can be given as

$\phi = \frac{180^o - \theta}{2} = (90^o - \theta/2)$



Note : ❑ Relation between linear velocity and angular velocity.

In vector form $\vec{v} = \vec{\omega}\times\vec{r}$

*S*ample problems based on velocity

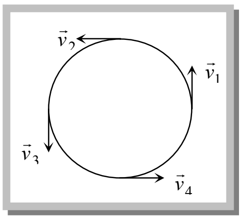
***Problem* 89.** If a particle moves in a circle describing equal angles in equal times, its velocity vector **[CPMT 1972, 74; JIPMER 1997]**

(a) Remains constant (b) Changes in magnitude

(c) Changes in direction (d) Changes both in magnitude and direction

Solution : (c) In uniform circular motion velocity vector changes in direction but its magnitude always remains constant.

$|\overrightarrow{v_1}| = |\overrightarrow{v_2}| = |\overrightarrow{v_3}| = |\overrightarrow{v_4}|$ = constant



***Problem* 90.** A body is whirled in a horizontal circle of radius 20 *cm*. It has angular velocity of 10 *rad/s*. What is its linear velocity at any point on circular path **[CBSE PMT 1996; JIPMER 2000]**

(a) 10 *m/s* (b) 2 *m/s* (c) 20 *m/s* (d) $\sqrt{2}$ *m/s*

Solution : (b) $v = r\omega = 0.2 \times 10 = 2\, m/s$

***Problem* 91.** The linear velocity of a point on the equator is nearly (radius of the earth is 6400 *km*)

(a) 800 *km/hr* (b) 1600 *km/hr* (c) 3200 *km/hr* (d) 6400 *km/hr*

Solution : (b) $v = rw = 6400\ km \times \frac{2\pi}{24}\frac{rad}{hr} = 1675\ km/hr = 1600\ km/hr$

***Problem* 92.** A particle moves along a circle with a uniform speed *v*. After it has made an angle of 60° its speed will be

(a) $v\sqrt{2}$ (b) $\frac{v}{\sqrt{2}}$ (c) $\frac{v}{\sqrt{3}}$ (d) v

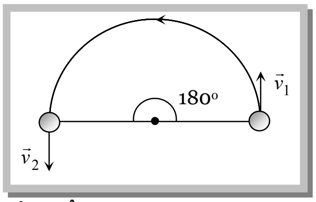
Solution : (d) Uniform speed means speed of the particle remains always constant.

***Problem* 93.** A particle is moving along a circular path of radius 2 *m* and with uniform speed of 5 ms^{-1}. What will be the change in velocity when the particle completes half of the revolution

(a) Zero (b) 10 ms^{-1} (c) $10\sqrt{2}\, ms^{-1}$ (d) $10/\sqrt{2}\, ms^{-1}$

Solution : (b) $\Delta v = 2v\sin\left(\frac{\theta}{2}\right) = 2 \times 5 \sin\left(\frac{180^o}{2}\right)$

$= 2 \times 5 \sin 90^o = 10\ m/s$



***Problem* 94.** What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$ **[Pb. PMT 2000]**

(a) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (b) $18\hat{i} + 13\hat{j} - 2\hat{k}$ (c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$

Solution : (b) $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{J} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$

$\vec{v} = (-24 + 6)i - (18 - 5)\hat{J} + (-18 + 20)\hat{k} = 18\,\hat{i} + 13\,\hat{J} - 2\hat{k}$

***Problem* 95.** A particle comes round circle of radius 1 *m* once. The time taken by it is 10 *sec*. The average velocity of motion is **[JIPMER 1999]**

(a) 0.2 π *m/s* (b) 2 π *m/s* (c) 2 *m/s* (d) Zero

Solution : (d) In complete revolution total displacement becomes zero. So the average velocity will be zero.

Problem 96. Two particles of mass M and m are moving in a circle of radii R and r. If their time-periods are same, what will be the ratio of their linear velocities **[CBSE PMT 2001]**

(a) $MR : mr$ (b) $M : m$ (c) $R : r$ (d) 1 : 1

Solution : (c) $\frac{v_1}{v_2} = \frac{r_1\omega_1}{r_2\omega_2}$. Time periods are equal *i.e.* $\omega_1 = \omega_2 \therefore \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{R}{r}$

(5) **Time period (T) :** In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.

(i) Units : second. (ii) Dimension : $[M^oL^oT]$

(iii) Time period of second's hand of watch = 60 *second*. (iv) Time period of minute's hand of watch = 60 *minute* (v) Time period of hour's hand of watch = 12 *hour*

(6) **Frequency (n) :** In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.

(i) Units : s^{-1} or hertz (Hz). (ii) Dimension : $[M^oL^oT^{-1}]$

Note : ❑ Relation between time period and frequency : If n is the frequency of revolution of an object in circular motion, then the object completes n revolutions in 1 second. Therefore, the object will complete one revolution in $1/n$ second.

$$\therefore T = 1/n$$

❑ Relation between angular velocity, frequency and time period : Consider a point object describing a uniform circular motion with frequency n and time period T. When the object completes one revolution, the angle traced at its axis of circular motion is 2π radians. It means, when time $t = T$, $\theta = 2\pi$ radians. Hence, angular velocity $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi n$ $(\because T = 1/n)$

$$\omega = \frac{2\pi}{T} = 2\pi n$$

❑ If two particles are moving on same circle or different coplanar concentric circles in same direction with different uniform angular speeds ω_A and ω_B respectively, the angular velocity of B relative to A will be

$$\omega_{rel} = \omega_B - \omega_A$$

So the time taken by one to complete one revolution around O with respect to the other (i.e., time in which B complete one revolution around O with respect to the other (*i.e.*, time in which B completes one more or less revolution around O than A)

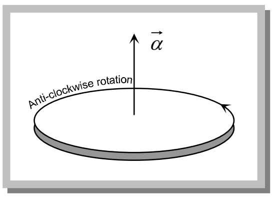
$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1T_2}{T_1 - T_2} \quad \left[\text{as } T = \frac{2\pi}{\omega}\right]$$

Special case : If $\omega_B = \omega_A, \omega_{rel} = 0$ and so $T = \infty$., particles will maintain their position relative to each other. This is what actually happens in case of geostationary satellite ($\omega_1 = \omega_2$ = constant)

(7) **Angular acceleration (α) :** Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) If $\Delta\omega$ be the change in angular velocity of the object in time interval t and $t + \Delta t$, while moving on a circular path, then angular acceleration of the object will be

$$\alpha = \underset{\Delta t \to 0}{Lt} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$



(ii) Units : $rad.\ s^{-2}$ (iii) Dimension : $[M^oL^oT^{-2}]$

(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since ω is constant so $\alpha = \frac{d\omega}{dt} = 0$

(vi) For non-uniform circular motion $\alpha \neq 0$

Note : ❑ Relation between linear (tangential) acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

❑ For uniform circular motion angular acceleration is zero, so tangential acceleration also is equal to zero.

❑ For non-uniform circular motion $a \neq 0$ (because $\alpha \neq 0$).

Sample problems based on angular acceleration

***Problem* 97.** A body is revolving with a uniform speed v in a circle of radius r. The angular acceleration of the body is

(a) $\frac{v}{r}$ (b) Zero (c) $\frac{v^2}{r}$ along the radius and towards the centre (d) $\frac{v^2}{r}$ along the radius and away from the centre

Solution : (b) In uniform circular motion ω constant so $\alpha = \frac{d\omega}{dt} = 0$

***Problem* 98.** The linear acceleration of a car is $10 m/s^2$. If the wheels of the car have a diameter of $1m$, the angular acceleration of the wheels will be

(a) $10\ rad/sec^2$ (b) $20\ rad/sec^2$ (c) $1\ rad/sec^2$ (d) $2\ rad/sec^2$

Solution : (b) $\text{Angular acceleration} = \frac{\text{linear acceleration}}{\text{radius}} = \frac{10}{0.5} = 20\ rad/sec^2$

***Problem* 99.** The angular speed of a motor increases from 600 *rpm* to 1200 *rpm* in 10 *s*. What is the angular acceleration of the motor

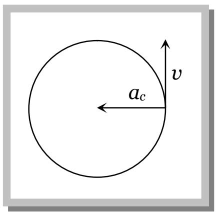
(a) $600\ rad\ sec^{-2}$ (b) $60\pi\ rad\ sec^{-2}$ (c) $60\ rad\ sec^{-2}$ (d) $2\pi\ rad\ sec^{-2}$

Solution : (d) $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi(1200 - 600)}{10 \times 60}\frac{rad}{sec^2} = 2\pi\ rad/sec^2$

3.10 Centripetal Acceleration

(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

(2) It always acts on the object along the radius towards the centre of the circular path.



(3) Magnitude of centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r = 4\pi n^2 r = \frac{4\pi^2}{T^2} r$

(4) Direction of centripetal acceleration : It is always the same as that of $\Delta\vec{v}$. When Δt decreases, $\Delta\theta$ also decreases. Due to which $\Delta\vec{v}$ becomes more and more perpendicular to $\vec{v}$. When $\Delta t \to 0$, $\Delta\vec{v}$ becomes perpendicular to the velocity vector. As the velocity vector of the particle at an instant acts along the tangent to the circular path, therefore $\Delta\vec{v}$ and hence the centripetal acceleration vector acts along the radius of the circular path at that point and is directed towards the centre of the circular path.

Sample problems based on centripetal acceleration

***Problem* 100.** If a cycle wheel of radius 4 *m* completes one revolution in two seconds. Then acceleration of the cycle will be

(a) $\pi^2 m/s^2$ (b) $2\pi^2 m/s^2$ (c) $4\pi^2 m/s^2$ (d) $8\pi\, m/s^2$ **[Pb. PMT 2001]**

Solution : (c) Given $r = 4\,m$ and $T = 2$ seconds.

$$\therefore a_c = \frac{4\pi^2}{T^2} r = \frac{4\pi^2}{(2)^2} 4 = 4\pi^2\, m/s^2$$

***Problem* 101.** A stone is tied to one end of a spring 50 *cm* long is whirled in a horizontal circle with a constant speed. If the stone makes 10 revolutions in 20 *s*, what is the magnitude of acceleration of the stone **[Pb. PMT 2000]**

(a) 493 *cm/sec*^{2} (b) 720 *cm/sec*^{2} (c) 860 *cm/sec*^{2} (d) 990 *cm/sec*^{2}

Solution : (a) Time period $= \frac{\text{Total time}}{\text{No. of revolution}} = \frac{20}{10} = 2\,sec$

$$\therefore a_c = \frac{4\pi^2}{T^2} . r = \frac{4\pi^2}{(2)^2} \times (1/2) m/s^2 = 4.93\, m/s^2 = 493\, cm/s^2$$

***Problem* 102.** A particle moves with a constant speed v along a circular path of radius r and completes the circle in time T. What is the acceleration of the particle **[Orissa JEE 2002]**

(a) mg (b) $\frac{2\pi v}{T}$ (c) $\frac{\pi r^2}{T}$ (d) $\frac{\pi v^2}{T}$

Solution : (b) $a_c = \frac{v^2}{r} = \omega^2 r = v\omega = v\left(\frac{2\pi}{T}\right) = \frac{2\pi v}{T}$

***Problem* 103.** If the speed of revolution of a particle on the circumference of a circle and the speed gained in falling through a distance equal to half the radius are equal, then the centripetal acceleration will be

(a) $\frac{g}{2}$ (b) $\frac{g}{4}$ (c) $\frac{g}{3}$ (d) g

Solution : (d) Speed gain by body falling through a distance h is equal to $v = \sqrt{2gh} = \sqrt{2g\frac{r}{2}}$ [As $h = \frac{r}{2}$ given]

$$\Rightarrow v = \sqrt{gr} \Rightarrow \frac{v^2}{r} = g$$

***Problem* 104.** Two cars going round curve with speeds one at 90 *km/h* and other at 15 *km/h*. Each car experiences same acceleration. The radii of curves are in the ratio of **[EAMCET (Med.) 1998]**

(a) 4 : 1 (b) 2 : 1 (c) 16 : 1 (d) 36 : 1

Solution : (d) Centripetal acceleration $= \frac{v_1^2}{r_1} = \frac{v_2^2}{r_2}$ (given)

$$\therefore \frac{r_1}{r_2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{90}{15}\right)^2 = \frac{36}{1}$$

***Problem* 105.** A wheel of radius $0.20m$ is accelerated from rest with an angular acceleration of $1\, rad/s^2$. After a rotation of 90^o the radial acceleration of a particle on its rim will be

(a) $\pi\, m/s^2$ (b) $0.5\,\pi\, m/s^2$ (c) $2.0\pi\, m/s^2$ (d) $0.2\,\pi\, m/s^2$

Solution : (d) From the equation of motion

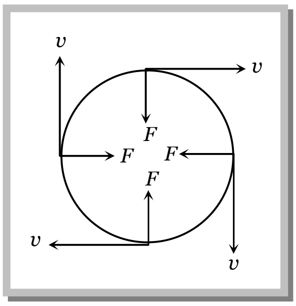
Angular speed acquired by the wheel, $\omega_2^2 = \omega_1^2 + 2\alpha\theta = 0 + 2\times 1 \times \frac{\pi}{2} \Rightarrow \omega_2^2 = \pi$

Now radial acceleration $\omega^2 r = \pi \times 0.2 = 0.2\pi\, m/s^2$

3.11 Centripetal Force

According to Newton's first law of motion, whenever a body moves in a straight line with uniform velocity, no force is required to maintain this velocity. But when a body moves along a circular path with uniform speed, its direction changes continuously *i.e.* velocity keeps on changing on account of a change in direction. According to Newton's second law of motion, a change in the direction of motion of the body can take place only if some external force acts on the body.

Due to inertia, at every point of the circular path; the body tends to move along the tangent to the circular path at that point (in figure). Since every body has directional inertia, a velocity cannot change by itself and as such we have to apply a force. But this force should be such that it changes the direction of velocity and not its magnitude. This is possible only if the force acts perpendicular to the direction of velocity. Because the velocity is along the tangent, this force must be along the radius (because the radius of a circle at any point is perpendicular to the tangent at that point). Further, as this force is to move the body in a circular path, it must acts towards the centre. This centre-seeking force is called the centripetal force.



Hence, centripetal force is that force which is required to move a body in a circular path with uniform speed. The force acts on the body along the radius and towards centre.

(1) Formulae for centripetal force : $F = \frac{mv^2}{r} = m\omega^2 r = m\,4\pi^2 n^2 r = \frac{m 4\pi^2 r}{T^2}$

(2) Centripetal force in different situation

Situation	Centripetal Force
A particle tied to a string and whirled in a horizontal circle	Tension in the string
Vehicle taking a turn on a level road	Frictional force exerted by the road on the tyres
A vehicle on a speed breaker	Weight of the body or a component of weight
Revolution of earth around the sun	Gravitational force exerted by the sun
Electron revolving around the nucleus in an atom	Coulomb attraction exerted by the protons in the nucleus

A charged particle describing a circular path in a magnetic field	Magnetic force exerted by the agent that sets up the magnetic field

3.12 Centrifugal Force

It is an imaginary force due to incorporated effects of inertia. When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer *A* who is not sharing the motion along the circular path, the body appears to fly off tangential at the point of release. To another observer *B*, who is sharing the motion along the circular path (*i.e.*, the observer *B* is also rotating with the body with the same velocity), the body appears to be stationary before it is released. When the body is released, it appears to *B*, as if it has been thrown off along the radius away from the centre by some force. In reality no force is actually seen to act on the body. In absence of any real force the body tends to continue its motion in a straight line due to its inertia. The observer *A* easily relates this events to be due to inertia but since the inertia of both the observer *B* and the body is same, the observer *B* can not relate the above happening to inertia. When the centripetal force ceases to act on the body, the body leaves its circular path and continues to moves in its straight-line motion but to observer *B* it appears that a real force has actually acted on the body and is responsible for throwing the body radially out-words. This imaginary force is given a name to explain the effects on inertia to the observer who is sharing the circular motion of the body. This inertial force is called centrifugal force. Thus centrifugal force is a fictitious force which has significance only in a rotating frame of reference.

~~Sample problems based on centripetal and centrifugal force~~

***Problem* 106.** A ball of mass 0.1 *kg* is whirled in a horizontal circle of radius 1 *m* by means of a string at an initial speed of 10 *r.p.m.* Keeping the radius constant, the tension in the string is reduced to one quarter of its initial value. The new speed is

(a) 5 *r.p.m.* (b) 10 *r.p.m.* (c) 20 *r.p.m.* (d) 14 *r.p.m.*

Solution : (a) Tension in the string $T = m\omega^2 r = m 4\pi^2 n^2 r$

$T \propto n^2$ or $n \propto \sqrt{T}$ [As *m* and *r* are constant]

$\therefore \frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{T/4}{T}} \Rightarrow n_2 = \frac{n_1}{2} = \frac{10}{2} = 5\, rpm$

***Problem* 107.** A cylindrical vessel partially filled with water is rotated about its vertical central axis. It's surface will **[RPET 2000]**

(a) Rise equally (b) Rise from the sides (c) Rise from the middle (d) Lowered equally

Solution : (b) Due to the centrifugal force.

***Problem* 108.** A proton of mass 1.6×10^{-27} *kg* goes round in a circular orbit of radius 0.10 *m* under a centripetal force of 4×10^{-13} *N*. then the frequency of revolution of the proton is about **[Kerala PMT 2002]**

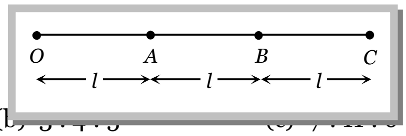
(a) 0.08×10^8 *cycles per sec* (b) 4×10^8 *cycles per* sec

(c) 8×10^8 *cycles per sec* (d) 12×10^8 *cycles per sec*

Solution : (a) $F = 4 \times 10^{-13}\, N$; $m = 1.6 \times 10^{-27}\, kg$; $r = 0.1\, m$

Centripetal force $F = m 4\pi^2 n^2 r \therefore n = \sqrt{\frac{F}{4m\pi^2 r}} = 8 \times 10^6\, cycles/\sec = 0.08 \times 10^8\, cycle/\sec$.

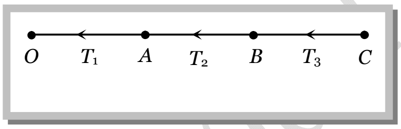
***Problem* 109.** Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is



(a) 3 : 5 : 7 (b) 3 : 4 : 5 (c) 7 : 11 : 6 (d) 3 : 5 : 6

Solution : (d) Let the angular speed of the thread is ω

For particle '*C*' $\Rightarrow T_3 = m\omega^2 3l$



For particle '*B*' $T_2 - T_3 = m\omega^2 2l \Rightarrow T_2 = m\omega^2 5l$

For particle '*C*' $T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = m\omega^2 6l$

$\therefore T_3 : T_2 : T_1 = 3 : 5 : 6$

***Problem* 110.** A stone of mass 1 *kg* tied to the end of a string of length 1 *m*, is whirled in a horizontal circle with a uniform angular velocity of 2 *rad/s*. The tension of the string is (in *N*) **[KCET 1998]**

(a) 2 (b) $\frac{1}{3}$ (c) 4 (d) $\frac{1}{4}$

Solution : (c) $T = m\omega^2 r = 1 \times (2)^2 \times (1) = 4$ *Newton*

***Problem* 111.** A cord can bear a maximum force of 100 *N* without breaking. A body of mass 1 *kg* tied to one end of a cord of length 1 *m* is revolved in a horizontal plane. What is the maximum linear speed of the body so that the cord does not break

(a) 10 *m/s* (b) 20 *m/s* (c) 25 *m/s* (d) 30 *m/s*

Solution : (a) Tension in cord appears due to centrifugal force $T = \frac{mv^2}{r}$ and for critical condition this tension will be equal to breaking force (100 *N*) $\therefore \frac{mv_{max}^2}{r} = 100 \Rightarrow v_{max}^2 = \frac{100 \times 1}{1} \Rightarrow v_{max} = 10\ m/s$

***Problem* 112.** A mass is supported on a frictionless horizontal surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω_0. If the length of the string and angular velocity are doubled, the tension in the string which was initially T_0 is now **[AIIMS 1985]**

(a) T_0 (b) $T_0/2$ (c) $4T_0$ (d) $8T_0$

Solution : (d) $T = m\omega^2 l \therefore \frac{T_2}{T_1} = \left(\frac{\omega_2}{\omega_1}\right)^2 \left(\frac{l_2}{l_1}\right) \Rightarrow \frac{T_2}{T_0} = \left(\frac{2\omega}{\omega}\right)^2 \left(\frac{2l}{l}\right) \Rightarrow T_2 = 8T_0$

***Problem* 113.** A stone is rotated steadily in a horizontal circle with a period *T* by a string of length *l*. If the tension in the string is kept constant and *l* increases by 1%, what is the percentage change in *T*

(a) 1% (b) 0.5% (c) 2% (d) 0.25%

Solution : (b) Tension $= \frac{m 4\pi^2 l}{T^2} \therefore l \propto T^2$ or $T \propto \sqrt{l}$ [Tension and mass are constant]

Percentage change in Time period $= \frac{1}{2}$ (percentage change in length) [If % change is very small]

$$= \frac{1}{2}(1\%) = 0.5\%$$

***Problem* 114.** If mass speed and radius of rotation of a body moving in a circular path are all increased by 50%, the necessary force required to maintain the body moving in the circular path will have to be increased by

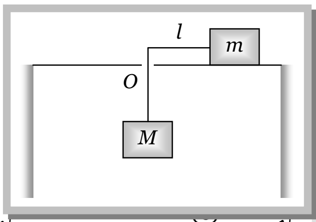
(a) 225% (b) 125% (c) 150% (d) 100%

Solution : (b) Centripetal force $F = \frac{m v^2}{r}$

If m, v and r are increased by 50% then let new force $F' = \frac{\left(m + \frac{m}{2}\right)\left(v + \frac{v}{2}\right)^2}{\left(r + \frac{r}{2}\right)} = \frac{9}{4}\frac{m v^2}{r} = \frac{9}{4}F$

Percentage increase in force $\frac{\Delta F}{F} \times 100 = \frac{F' - F}{F} \times 100\% = \frac{500}{4}\% = 125\%$

Problem 115. Two masses m and M are connected by a light string that passes through a smooth hole O at the centre of a table. Mass m lies on the table and M hangs vertically. m is moved round in a horizontal circle with O as the centre. If l is the length of the string from O to m then the frequency with which m should revolve so that M remains stationary is



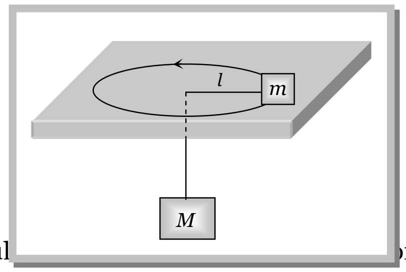
(a) $\frac{1}{2\pi}\sqrt{\frac{Mg}{m\,l}}$ (b) $\frac{1}{\pi}\sqrt{\frac{[…]}{m\,l}}$ (c) $[…]\frac{}{2\pi}\sqrt{\frac{l}{Mg}}$ (d) $\frac{1}{\pi}\sqrt{\frac{m\,l}{Mg}}$

Solution : (a) 'm' Mass performs uniform circular motion on the table. Let n is the frequency of revolution then centrifugal force $= m\,4\pi^2 n^2 l$

For equilibrium this force will be equal to weight Mg

$m\,4\pi^2 n^2 l = Mg$

$\therefore n = \frac{1}{2\pi}\sqrt{\frac{Mg}{m\,l}}$



Problem 116. A particle of mass M moves with constant speed along a circul[…]n of a force F. Its speed is **[MP PMT 2002]**

(a) $\sqrt{\frac{r\,F}{m}}$ (b) $\sqrt{\frac{F}{r}}$ (c) $\sqrt{F\,m\,r}$ (d) $\sqrt{\frac{F}{m\,r}}$

Solution : (a) Centripetal force $F = \frac{m v^2}{r}$ $\therefore v = \sqrt{\frac{r\,F}{m}}$

Problem 117. In an atom for the electron to revolve around the nucleus, the necessary centripetal force is obtained from the following force exerted by the nucleus on the electron

(a) Nuclear force (b) Gravitational force (c) Magnetic force (d) Electrostatic force

Solution : (d)

Problem 118. A motor cycle driver doubles its velocity when he is having a turn. The force exerted outwardly will be **[AFMC 2002]**

(a) Double (b) Half (c) 4 times (d) $\frac{1}{4}$ times

Solution : (c) $F = \frac{m v^2}{r}$ $\therefore F \propto v^2$ or $\frac{F_2}{F_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{2v}{v}\right)^2 = 4 \Rightarrow F_2 = 4F_1$

Problem 119. A bottle of soda water is grasped by the neck and swing briskly in a vertical circle. Near which portion of the bottle do the bubbles collect

(a) Near the bottom (b) In the middle of the bottle

(c) Near the neck (d) Uniformly distributed in the bottle

Solution : (c) Due to the lightness of the gas bubble they feel less centrifugal force so they get collect near the neck of the bottle. They collect near the centre of circular motion *i.e.* near the neck of the bottle.

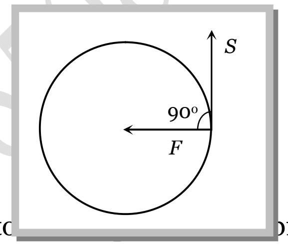
Problem **120.** A body is performing circular motion. An observer O_1 is sitting at the centre of the circle and another observer O_2 is sitting on the body. The centrifugal force is experienced by the observer

(a) O_1 only (b) O_2 only (c) Both by O_1 and O_2 (d) None of these

Solution : (b) Centrifugal force is a pseudo force, which is experienced only by that observer who is attached with the body performing circular motion.

3.13 Work done by Centripetal Force

The work done by centripetal force is always zero as it is perpendicular to velocity and hence instantaneous displacement.



Work done = Increment in kinetic energy of revolving body

Work done = 0

Also $W = \vec{F}.\vec{S} = F \cdot S\cos\theta$

$= F{\cdot}S \cos 90^o = 0$

Example : (i) When an electron revolve around the nucleus in hydrogen atc [illegible] bit, it neither absorb nor emit any energy means its energy remains constant.

(ii) When a satellite established once in a orbit around the earth and it starts revolving with particular speed, then no fuel is required for its circular motion.

Sample problem based on work done

Problem **121.** A particle does uniform circular motion in a horizontal plane. The radius of the circle is 20 *cm*. The centripetal force acting on the particle is 10 *N*. It's kinetic energy is

(a) 0.1 *Joule* (b) 0.2 *Joule* (c) 2.0 *Joule* (d) 1.0 *Joule*

Solution : (d) $\frac{mv^2}{r} = 10\ N$ (given) $\Rightarrow mv^2 = 10\times r = 10\times 0.2 = 2$

Kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}(2) = 1\ Joule$.

Problem **122.** A body of mass 100 *g* is rotating in a circular path of radius *r* with constant velocity. The work done in one complete revolution is **[AFMC 1998]**

(a) 100*r Joule* (b) (*r*/100) *Joule* (c) (100/*r*) *Joule* (d) Zero

Solution : (d) Because in uniform circular motion work done by the centripetal force is always zero.

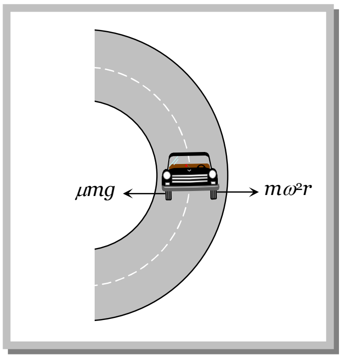
Problem **123.** A particle of mass *m* is describing a circular path of radius *r* with uniform speed. If *L* is the angular momentum of the particle about the axis of the circle, the kinetic energy of the particle is given by **[CPMT 1995]**

(a) L^2/mr^2 (b) $L^2/2mr^2$ (c) $2L^2/mr^2$ (d) mr^2L

Solution : (b) Rotational kinetic energy $E = \frac{L^2}{2I} = \frac{L^2}{2mr^2}$ (As for a particle $I = mr^2$)

3.14 Skidding of Vehicle on a Level Road

When a vehicle turns on a circular path it requires centripetal force.



If friction provides this centripetal force then vehicle can move in circular path safely if

Friction force ≥ Required centripetal force

$$\mu mg \geq \frac{mv^2}{r}$$

$$\therefore \qquad v_{safe} \leq \sqrt{\mu rg}$$

This is the maximum speed by which vehicle can turn in a circular path of radius *r*, where coefficient of friction between the road and tyre is μ.

Sample problem based on skidding of vehicle on a level road

***Problem* 124.** Find the maximum velocity for overturn for a car moved on a circular track of radius $100\,m$. The coefficient of friction between the road and tyre is 0.2

(a) $0.14\ m/s$ (b) $140\ m/s$ (c) $1.4\ km/s$ (d) $14\ m/s$

Solution : (d) $v_{\max} = \sqrt{\mu r g} = \sqrt{0.2 \times 100 \times 10} = 10\sqrt{2} = 14\ m/s$

***Problem* 125.** When the road is dry and the coefficient of friction is μ, the maximum speed of a car in a circular path is $10\ m/s$. If the road becomes wet and $\mu' = \frac{\mu}{2}$, what is the maximum speed permitted

(a) $5\ m/s$ (b) $10\ m/s$ (c) $10\sqrt{2}\ m/s$ (d) $5\sqrt{2}\ m/s$

Solution : (d) $v \propto \sqrt{\mu} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{\mu_2}{\mu_1}} = \sqrt{\frac{\mu/2}{\mu}} = \frac{1}{\sqrt{2}} \Rightarrow v_2 = \frac{1}{\sqrt{2}} v_1 \Rightarrow v_2 = \frac{10}{\sqrt{2}} = 5\sqrt{2}\ m/s$

***Problem* 126.** The coefficient of friction between the tyres and the road is 0.25. The maximum speed with which a car can be driven round a curve of radius 40 *m* with skidding is (assume $g = 10\ ms^{-2}$)

(a) $40\ ms^{-1}$ (b) $20\ ms^{-1}$ (c) $15\ ms^{-1}$ (d) $10\ ms^{-1}$

Solution : (d) $v_{\max} = \sqrt{\mu r g} = \sqrt{0.25 \times 40 \times 10} = 10\ m/s$

3.15 Skidding of Object on a Rotating Platform

On a rotating platform, to avoid the skidding of an object (mass *m*) placed at a distance *r* from axis of rotation, the centripetal force should be provided by force of friction.

Centripetal force = Force of friction

$$m\omega^2 r = \mu mg$$

$$\therefore \qquad \omega_{\max} = \sqrt{(\mu g/r)},$$

Hence maximum angular velocity of rotation of the platform is $\sqrt{(\mu g/r)}$, so that object will not skid on it.

3.16 Bending of a Cyclist

A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track, while going round a curve. Consider a cyclist of weight *mg* taking a turn of radius *r* with velocity *v*. In order to provide the necessary centripetal force, the cyclist leans through angle θ inwards as shown in figure.

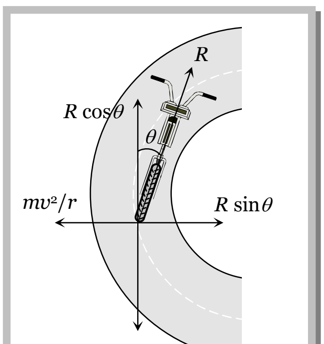
The cyclist is under the action of the following forces :

The weight *mg* acting vertically downward at the centre of gravity of cycle and the cyclist.

The reaction *R* of the ground on cyclist. It will act along a line-making angle θ with the vertical.

The vertical component $R\cos\theta$ of the normal reaction *R* will balance the weight of the cyclist, while the horizontal component $R\sin\theta$ will provide the necessary centripetal force to the cyclist.

$$R\sin\theta = \frac{mv^2}{r} \qquad .....(i)$$



and $R\cos\theta = mg$ .....(ii)

Dividing equation (i) by (ii), we have

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2/r}{mg}$$

or $\tan\theta = \frac{v^2}{rg}$ ....(iii)

Therefore, the cyclist should bend through an angle $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

It follows that the angle through which cyclist should bend will be greater, if

(i) The radius of the curve is small *i.e.* the curve is sharper

(ii) The velocity of the cyclist is large.

Note : ❑ For the same reasons, an ice skater or an aeroplane has to bend inwards, while taking a turn.

Sample problem based on bending of cyclist

***Problem* 127.** A boy on a cycle pedals around a circle of 20 metres radius at a speed of 20 *metres/sec*. The combined mass of the boy and the cycle is $90\,kg$. The angle that the cycle makes with the vertical so that it may not fall is $(g = 9.8\ m/\sec^2)$ **[MP PMT 1995]**

(a) 60.25^o (b) 63.90^o (c) 26.12^o (d) 30.00^o

Solution : (b) $r = 20\,m,\ v = 20\,m/s,\ m = 90\,kg,\ g = 9.8\,m/s^2$ (given)

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20\times 20}{20\times 10}\right) = \tan^{-1}(2) = 63.90°$$

***Problem* 128.** If a cyclist moving with a speed of $4.9\ m/s$ on a level road can take a sharp circular turn of radius $4m$, then coefficient of friction between the cycle tyres and road is

(a) 0.41 (b) 0.51 (c) 0.71 (d) 0.61

Solution : (d) $v = 4.9\,m/s,\ r = 4\,m$ and $g = 9.8\,m/s^2$ (given)

$$\mu = \frac{v^2}{rg} = \frac{4.9\times 4.9}{4\times 9.8} = 0.61$$

***Problem* 129.** A cyclist taking turn bends inwards while a car passenger taking same turn is thrown outwards. The reason is **[NCERT 1972]**

(a) Car is heavier than cycle

(b) Car has four wheels while cycle has only two

(c) Difference in the speed of the two

(d) Cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force

Solution : (d)

3.17 Banking of a Road

For getting a centripetal force cyclist bend towards the centre of circular path but it is not possible in case of four wheelers.

Therefore, outer bed of the road is raised so that a vehicle moving on it gets automatically inclined towards the centre.

In the figure (A) shown reaction R is resolved into two components, the component $R\cos\theta$ balances weight of vehicle

$\therefore \qquad R\cos\theta = mg$ ......(i)

and the horizontal component $R\sin\theta$ provides necessary centripetal force as it is directed towards centre of desired circle

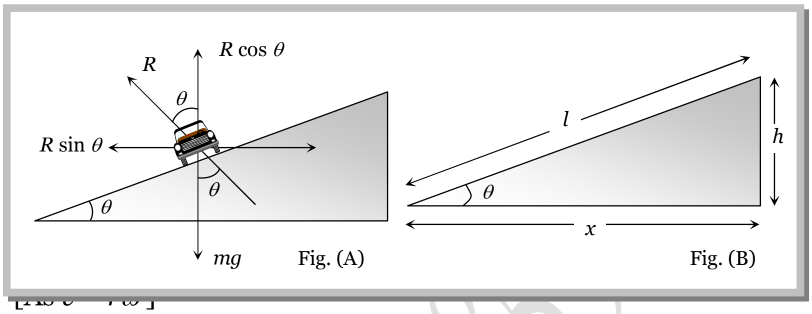


Fig. (A) Fig. (B)

Thus $R\sin\theta = \dfrac{mv^2}{r}$ ......(ii)

Dividing (ii) by (i), we have

$\tan\theta = \dfrac{v^2}{rg}$ ...... (iii)

or $\tan\theta = \dfrac{\omega^2 r}{g} = \dfrac{v\omega}{rg}$ ...... (iv) [As $v = r\omega$]

If l = width of the road, h = height of the outer edge from the ground level then from the figure (B)

$\tan\theta = \dfrac{h}{x} = \dfrac{h}{l}$ ....(v) [since θ is very small]

From equation (iii), (iv) and (v) $\tan\theta = \dfrac{v^2}{rg} = \dfrac{\omega^2 r}{g} = \dfrac{v\omega}{rg} = \dfrac{h}{l}$

Note : ❑ If friction is also present between the tyres and road then $\dfrac{v^2}{rg} = \dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}$

❑ Maximum safe speed on a banked frictional road $v = \sqrt{\dfrac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$

Sample problems based on banking of a road

***Problem* 130.** For traffic moving at $60\ km/hr$ along a circular track of radius $0.1\ km$, the correct angle of banking is

[MNR 1993]

(a) $\dfrac{(60)^2}{0.1}$ (b) $\tan^{-1}\left[\dfrac{(50/3)^2}{100\times 9.8}\right]$ (c) $\tan^{-1}\left[\dfrac{100\times 9.8}{(50/3)^2}\right]$ (d) $\tan^{-1}\sqrt{60\times 0.1\times 9.8}$

Solution : (b) $v = 60\ km/hr = \dfrac{50}{3} m/s$, $r = 0.1 km = 100\ m$, $g = 9.8\ m/s^2$ (given)

Angle of banking $\tan\theta = \dfrac{v^2}{rg}$ or $\theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right) = \tan^{-1}\left[\dfrac{(50/3)^2}{100\times 9.8}\right]$

***Problem* 131.** A vehicle is moving with a velocity v on a curved road of width b and radius of curvature R. For counteracting the centrifugal force on the vehicle, the difference in elevation required in between the outer and inner edges of the road is

(a) $\dfrac{v^2 b}{Rg}$ (b) $\dfrac{rb}{Rg}$ (c) $\dfrac{vb^2}{Rg}$ (d) $\dfrac{vb}{R^2 g}$

Solution : (a) For Banking of road $\tan\theta = \dfrac{v^2}{rg}$ and $\tan\theta = \dfrac{h}{l}$

$\therefore \dfrac{v^2}{rg} = \dfrac{h}{l} \Rightarrow h = \dfrac{v^2 l}{rg} = \dfrac{v^2 b}{Rg}$ [As $l = b$ and $r = R$ given]

***Problem* 132.** The radius of curvature of a road at a certain turn is $50m$. The width of the road is $10m$ and its outer edge is $1.5m$ higher than the inner edge. The safe speed for such an inclination will be

(a) $6.5\ m/s$ (b) $8.6\ m/s$ (c) $8\ m/s$ (d) $10\ m/s$

Solution : (b) $h = 1.5\,m,\ r = 50\,m,\ l = 10\,m,\ g = 10\,m/s^2$ (given)

$$\frac{v^2}{rg} = \frac{h}{l} \Rightarrow v = \sqrt{\frac{hrg}{l}} = \sqrt{\frac{1.5 \times 50 \times 10}{10}} = 8.6\,m/s$$

***Problem* 133.** Keeping the banking angle same to increase the maximum speed with which a car can travel on a curved road by 10%, the radius of curvature of road has to be changed from $20m$ to **[EAMCET 1991]**

(a) $16m$ (b) $18m$ (c) $24.25m$ (d) $30.5\ m$

Solution : (c) $\tan\theta = \frac{v^2}{rg} \Rightarrow r \propto v^2$ (if θ is constant)

$$\frac{r_2}{r_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{1.1v}{v}\right)^2 = 1.21 \Rightarrow r_2 = 1.21 \times r_1 = 1.21 \times 20 = 24.2\ m$$

***Problem* 134.** The slope of the smooth banked horizontal road is p. If the radius of the curve be r, the maximum velocity with which a car can negotiate the curve is given by

(a) prg (b) $\sqrt{prg}$ (c) p/rg (d) $\sqrt{p/rg}$

Solution : (b) $\tan\theta = \frac{v^2}{rg} \Rightarrow p = \frac{v^2}{rg} \therefore v = \sqrt{prg}$

3.18 Overturning of Vehicle

When a car moves in a circular path with speed more than maximum speed then it overturns and it's inner wheel leaves the ground first

Weight of the car = mg

Speed of the car = v

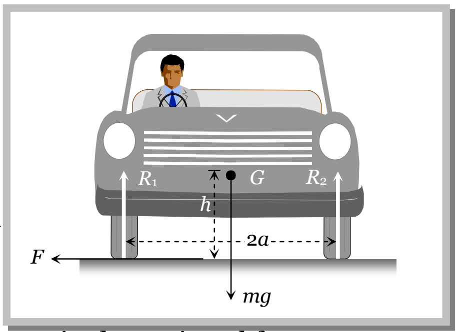
Radius of the circular path = r

Distance between the centre of wheels of the car = $2a$

Height of the centre of gravity (G) of the car from the road level = h

Reaction on the inner wheel of the car by the ground = R_1

Reaction on the outer wheel of the car by the ground = R_2



When a car move in a circular path, horizontal force F provides the required centripetal force

$$i.e.,\ F = \frac{mv^2}{R} \quad ....(i)$$

For rotational equilibrium, by taking the moment of forces R_1, R_2 and F about G

$$Fh + R_1 a = R_2 a \quad ....(ii)$$

As there is no vertical motion so $R_1 + R_2 = mg$ ....(iii)

By solving (i), (ii) and (iii)

$$R_1 = \frac{1}{2}M\left[g - \frac{v^2 h}{ra}\right] \quad ....(iv)$$

and

$$R_2 = \frac{1}{2}M\left[g + \frac{v^2 h}{ra}\right] \quad ....(v)$$

It is clear from equation (iv) that if v increases value of R_1 decreases and for $R_1 = 0$

$$\frac{v^2 h}{ra} = g \quad \text{or} \quad v = \sqrt{\frac{gra}{h}}$$

i.e. the maximum speed of a car without overturning on a flat road is given by $v = \sqrt{\frac{gra}{h}}$

Sample problems based on overturning of vehicle

***Problem* 135.** The distance between two rails is $1.5m$. The centre of gravity of the train at a height of $2m$ from the ground. The maximum speed of the train on a circular path of radius $120\,m$ can be

(a) $10.5\ m/s$ (b) $42\ m/s$ **(c)** $21\ m/s$ (d) $84\ m/s$

Solution : (c) Height of centre of gravity from the ground $h = 2m$, Acceleration due to gravity $g = 10\,m/s^2$,

Distance between two rails $2a = 1.5m$, Radius of circular path $r = 120\ m$ (given)

$$v_{\max} = \sqrt{\frac{gra}{h}} \Rightarrow v_{\max} = \sqrt{\frac{10 \times 120 \times 0.75}{2}} = 21.2\,m/s$$

***Problem* 136.** A car sometimes overturns while taking a turn. When it overturns, it is **[AFMC 1988]**

(a) The inner wheel which leaves the ground first

(b) The outer wheel which leaves the ground first

(c) Both the wheels leave the ground simultaneously

(d) Either wheel leaves the ground first

Solution : (a)

***Problem* 137.** A car is moving on a circular path and takes a turn. If R_1 and R_2 be the reactions on the inner and outer wheels respectively, then

(a) $R_1 = R_2$ (b) $R_1 < R_2$ (c) $R_1 > R_2$ (d) $R_1 \geq R_2$

Solution : (b) Reaction on inner wheel $R_1 = \frac{M}{2}\left[g - \frac{v^2 h}{ra}\right]$ and Reaction on outer wheel $R_2 = \frac{M}{2}\left[g + \frac{v^2 h}{ra}\right]$

$\therefore\ R_1 < R_2$.

***Problem* 138.** A train A runs from east to west and another train B of the same mass runs from west to east at the same speed along the equator. A presses the track with a force F_1 and B presses the track with a force F_2

(a) $F_1 > F_2$

(b) $F_1 < F_2$

(c) $F_1 = F_2$

(d) The information is insufficient to find the relation between F_1 and F_2

Solution : (a) We know that earth revolves about its own axis from west to east. Let its angular speed is ω_e and the angular speed of the train is ω_t

For train A : Net angular speed = $(\omega_e - \omega_t)$ because the sense of rotation of train is opposite to that of earth

So reaction of track $R_1 = F_1 = m\,g - m\,(\omega_e - \omega_t)^2 R$

For train B : Net angular speed = $(\omega_e + \omega_t)$ because the sense of rotation of train is same as that of earth

So reaction of track $R_2 = F_2 = m\,g - m\,(\omega_e + \omega_t)^2 R$

So it is clear that $F_1 > F_2$

3.19 Motion of Charged Particle in Magnetic Field

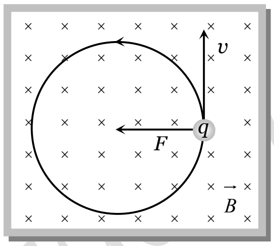
When a charged particle having mass m , charge q enters perpendicularly in a magnetic field B, with velocity v then it describes a circular path of radius r.

Because magnetic force (qvB) works in the perpendicular direction of v and it provides required centripetal force

Magnetic force = Centripetal force

$$qvB = \frac{mv^2}{r}$$

$\therefore$ radius of the circular path $r = \frac{mv}{qB}$

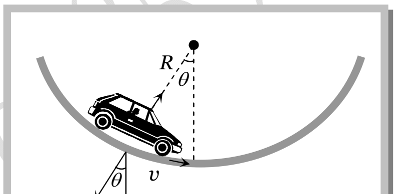


3.20 Reaction of Road on Car

(1) When car moves on a concave bridge then

Centripetal force = $R - mg\cos\theta = \frac{mv^2}{r}$

and reaction $R = mg\cos\theta + \frac{mv^2}{r}$

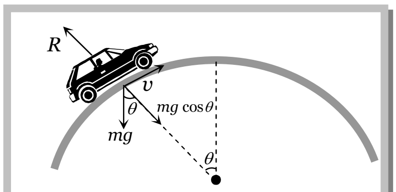


Concave bridge

(2) When car moves on a convex bridge

Centripetal force = $mg\cos\theta - R = \frac{mv^2}{r}$

and reaction $R = mg\cos\theta - \frac{mv^2}{r}$



Convex bridge

Sample problem based on reaction of road

Problem **139.** The road way bridge over a canal is in the form of an arc of a circle of radius $20m$. What is the minimum speed with which a car can cross the bridge without leaving contact with the ground at the highest point $(g = 9.8\ m/s^2)$

(a) $7\ m/s$ (b) $14\ m/s$ (c) $289\ m/s$ (d) $5\ m/s$

Solution : (b) At the highest point of the bridge for critical condition $mg - \frac{mv^2}{r} = 0 \Rightarrow \frac{mv^2}{r} = mg$

$\therefore v_{max} = \sqrt{gr} = \sqrt{9.8 \times 20} = \sqrt{196} = 14\ m/s$

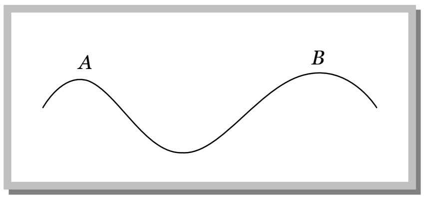
Problem **140.** A car moves at a constant speed on a road as shown in the figure. The normal force exerted by the road on the car is N_A and N_B when it is at the points A and B respectively

(a) $N_A = N_B$

(b) $N_A > N_B$

(c) $N_A < N_B$

(d) All possibilities are there



Solution : (c) From the formula $N = mg - \frac{mv^2}{r}$ $\therefore N \propto r$

As $r_A < r_B \quad \therefore \quad N_A < N_B$

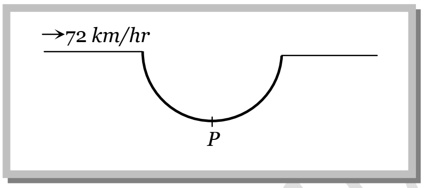
Problem 141. A car while travelling at a speed of $72\ km/hr$. Passes through a curved portion of road in the form of an arc of a radius $10m$. If the mass of the car is $500\ kg$ the reaction of the car at the lowest point P is

(a) $25\ kN$

(b) $50\ kN$

(c) $75\ kN$

(d) None of these



Solution : (a) $v = 72\frac{km}{h} = 20\ m/s,\quad r = 10\ m,\quad m = 500\ kg$ (given)

Reaction at lowest point $R = mg + \frac{mv^2}{r}$

$$= 500 \times 10 + \frac{500 \times (20)^2}{10} = 25000\ N = 25\ KN$$

3.21 Non-Uniform Circular Motion

If the speed of the particle in a horizontal circular motion changes with respect to time, then its motion is said to be non-uniform circular motion.

Consider a particle describing a circular path of radius r with centre at O. Let at an instant the particle be at P and $\vec{\upsilon}$ be its linear velocity and $\vec{\omega}$ be its angular velocity.

Then, $\vec{\upsilon} = \vec{\omega} \times \vec{r}$ .....(i)

Differentiating both sides of *w.r.t.* time t we have

$$\frac{d\vec{\upsilon}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} \quad .....(ii)$$

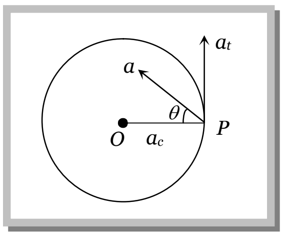
Here, $\frac{d\vec{v}}{dt} = \vec{a}$, (Resultant acceleration)

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\upsilon}$$

$\frac{d\vec{\omega}}{dt} = \vec{\alpha}$ (Angular acceleration)

$$\vec{a} = \vec{a}_t + \vec{a}_c \quad .....(iii)$$

$\frac{d\vec{r}}{dt} = \vec{\upsilon}$ (Linear velocity)



Thus the resultant acceleration of the particle at P has two component accelerations

(1) **Tangential acceleration :** $\vec{a_t} = \vec{\alpha} \times \vec{r}$

It acts along the tangent to the circular path at P in the plane of circular path.

According to right hand rule since $\vec{\alpha}$ and $\vec{r}$ are perpendicular to each other, therefore, the magnitude of tangential acceleration is given by

$$|\vec{a}_t| = |\vec{\alpha} \times \vec{r}| = \alpha r \sin 90^o = \alpha r.$$

(2) **Centripetal (Radial) acceleration** : $\overrightarrow{a_c} = \vec{\omega} \times \vec{v}$

It is also called centripetal acceleration of the particle at *P*.

It acts along the radius of the particle at *P*.

According to right hand rule since $\vec{\omega}$ and $\vec{\upsilon}$ are perpendicular to each other, therefore, the magnitude of centripetal acceleration is given by

$$|\vec{a}_c| = |\vec{\omega} \times \vec{\upsilon}| = \omega\upsilon \sin 90^o = \omega\upsilon = \omega(\omega r) = \omega^2 r = \upsilon^2 / r$$

(3) **Tangential and centripetal acceleration in different motions**

Centripetal acceleration	Tangential acceleration	Net acceleration	Type of motion
$a_c = 0$	$a_t = 0$	$a = 0$	Uniform translatory motion
$a_c = 0$	$a_t \neq 0$	$a = a_t$	Accelerated translatory motion
$a_c \neq 0$	$a_t = 0$	$a = a_c$	Uniform circular motion
$a_c \neq 0$	$a_t \neq 0$	$a = \sqrt{a_c^2 + a_t^2}$	Non-uniform circular motion

Note : ❑ Here a_t governs the magnitude of $\vec{v}$ while $\vec{a}_c$ its direction of motion.

(4) **Force :** In non-uniform circular motion the particle simultaneously possesses two forces

Centripetal force : $F_c = ma_c = \dfrac{mv^2}{r} = mr\omega^2$

Tangential force : $F_t = ma_t$

Net force : $F_{\text{net}} = ma = m\sqrt{a_c^2 + a_t^2}$

Note : ❑ In non-uniform circular motion work done by centripetal force will be zero since $\vec{F}_c \perp \vec{v}$

❑ In non uniform circular motion work done by tangential of force will not be zero since $F_t \neq 0$

❑ Rate of work done by net force in non-uniform circular = rate of work done by tangential force

i.e. $P = \dfrac{dW}{dt} = \vec{F}_t . \vec{v}$

Sample problems based on non-uniform circular motion

***Problem* 142.** The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered. It is given as *K.E.* = as^2 where a is a constant. The force acting on the particle is **[MNR 1992; JIPMER 2001, 2002]**

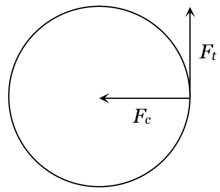
(a) $2a\dfrac{s^2}{R}$ (b) $2as\left(1 + \dfrac{s^2}{R^2}\right)^{1/2}$ (c) $2as$ (d) $2a\dfrac{R^2}{s}$

Solution : (b) In non-uniform circular motion two forces will work on a particle F_c and F_t

So the net force $F_{Net} = \sqrt{F_c^2 + F_t^2}$ ....(i)

Centripetal force $F_c = \dfrac{mv^2}{R} = \dfrac{2as^2}{R}$ ....(ii) [As kinetic energy $\dfrac{1}{2}mv^2 = as^2$ given]

Again from : $\frac{1}{2}mv^2 = as^2 \Rightarrow v^2 = \frac{2as^2}{m} \Rightarrow v = s\sqrt{\frac{2a}{m}}$



Tangential acceleration $a_t = \frac{dv}{dt} = \frac{dv}{ds}.\frac{ds}{dt} \Rightarrow a_t = \frac{d}{ds}\left[s\sqrt{\frac{2a}{m}}\right].v$

$a_t = v\sqrt{\frac{2a}{m}} = s\sqrt{\frac{2a}{m}}\sqrt{\frac{2a}{m}} = \frac{2as}{m}$

and $F_t = ma_t = 2as$ ....(iii)

Now substituting value of F_c and F_t in equation (i) $\therefore F_{Net} = \sqrt{\left(\frac{2as^2}{R}\right)^2 + (2as)^2} = 2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$

***Problem* 143.** A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t (c) $\frac{mk^4r^2t^5}{3}$ (d) Zero

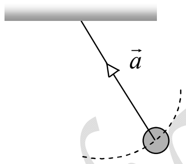
Solution : (b) $a_c = k^2rt^2 \Rightarrow \frac{v^2}{r} = k^2rt^2 \Rightarrow v^2 = k^2r^2t^2 \Rightarrow v = krt$

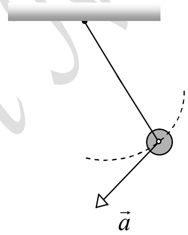
Tangential acceleration $a_t = \frac{dv}{dt} = kr$

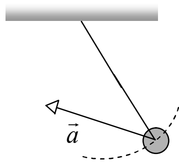
As centripetal force does not work in circular motion.

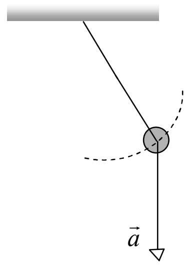
So power delivered by tangential force $P = F_tv = ma_tv = m(kr)\,krt = mk^2r^2t$

***Problem* 144.** A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in

(a) 

(b) 

(c) 

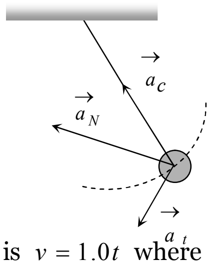
(d) 

Solution : (c) a_c = centripetal acceleration,

a_t = tangential acceleration,

a_N = net acceleration = Resultant of a_c and a_t

$a_N = \sqrt{a_c^2 + a_t^2}$



***Problem* 145.** The speed of a particle moving in a circle of radius $0.1m$ is $v = 1.0t$ where t is time in second. The resultant acceleration of the particle at $t = 5s$ will be

(a) $10\ m/s^2$ (b) $100\ m/s^2$ (c) $250\ m/s^2$ (d) $500\ m/s^2$

Solution : (c) $v = 1.0t \Rightarrow a_t = \frac{dv}{dt} = 1\,m/s^2$

and $a_c = \frac{v^2}{r} = \frac{(5)^2}{0.1} = 250\ m/s^2$ [At $t = 5$ sec, $v = 5\,m/s$]

$\therefore a_N = \sqrt{a_c^2 + a_t^2} = \sqrt{(250)^2 + 1^2} \Rightarrow a_N = 250\,m/s^2$ (approx.)

Problem 146. A particle moving along the circular path with a speed v and its speed increases by 'g' in one second. If the radius of the circular path be r, then the net acceleration of the particle is

(a) $\frac{v^2}{r}+g$ (b) $\frac{v^2}{r^2}+g^2$ (c) $\left[\frac{v^4}{r^2}+g^2\right]^{\frac{1}{2}}$ (d) $\left[\frac{v^2}{r}+g\right]^{\frac{1}{2}}$

Solution : (c) $a_t = g$ (given) and $a_c = \frac{v^2}{r}$ and $a_N = \sqrt{a_t^2+a_c^2} = \sqrt{\left(\frac{v^2}{r}\right)^2+g^2} = \sqrt{\frac{v^4}{r^2}+g^2}$

Problem 147. A car is moving with speed 30 m/sec on a circular path of radius 500 m. Its speed is increasing at the rate of $2 m/sec^2$. What is the acceleration of the car **[Roorkee 1982; RPET 1996; MH CET 2002; MP PMT 2003]**

(a) 2 m/s^2 (b) 2.7 m/s^2 (c) 1.8 m/s^2 (d) 9.8 m/s^2

Solution : (b) $a_t = 2 m/s^2$ and $a_c = \frac{v^2}{r} = \frac{30\times 30}{500} = 1.8\, m/s^2$ $\therefore\ a = \sqrt{a_t^2+a_c^2} = \sqrt{2^2+(1.8)^2} = 2.7 m/s^2$.

Problem 148. For a particle in circular motion the centripetal acceleration is **[CPMT 1998]**

(a) Less than its tangential acceleration (b) Equal to its tangential acceleration

(c) More than its tangential acceleration (d) May be more or less than its tangential acceleration

Solution : (d)

Problem 149. A particle is moving along a circular path of radius 3 *meter* in such a way that the distance travelled measured along the circumference is given by $S = \frac{t^2}{2}+\frac{t^3}{3}$. The acceleration of particle when $t = 2$ sec is

(a) 1.3 m/s² (b) 13 m/s² (c) 3 m/s² (d) 10 m/s²

Solution : (b) $s = \frac{t^2}{2}+\frac{t^3}{3} \Rightarrow v = \frac{ds}{dt} = t+t^2$ and $a_t = \frac{dv}{dt} = \frac{d}{dt}(t+t^2) = 1+2t$

At $t = 2$ *sec*, $v = 6$ m/s and $a_t = 5 m/s^2$, $a_c = \frac{v^2}{r} = \frac{36}{3} = 12\, m/s^2$

$a_N = \sqrt{a_c^2+a_t^2} = \sqrt{(12)^2+(5)^2} = 13\, m/s^2$.

3.22 Equations of Circular Motion

For accelerated motion	For retarded motion
$\omega_2 = \omega_1 + \alpha t$	$\omega_2 = \omega_1 - \alpha t$
$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$	$\theta = \omega_1 t - \frac{1}{2}\alpha t^2$
$\omega_2^2 = \omega_1^2 + 2\alpha\theta$	$\omega_2^2 = \omega_1^2 - 2\alpha\theta$
$\theta_n = \omega_1 + \frac{\alpha}{2}(2n-1)$	$\theta_n = \omega_1 - \frac{\alpha}{2}(2n-1)$

Where

ω_1 = Initial angular velocity of particle

ω_2 = Final angular velocity of particle

α = Angular acceleration of particle

θ = Angle covered by the particle in time t

θ_n = Angle covered by the particle in n^{th} second

~~Sample problems based on equation of circular motion~~

Problem 150. The angular velocity of a particle is given by $\omega = 1.5\,t - 3t^2 + 2$, the time when its angular acceleration ceases to be zero will be

(a) 25 sec (b) 0.25 sec (c) 12 sec (d) 1.2 sec

Solution : (b) $\omega = 1.5\,t - 3t^2 + 2$ and $\alpha = \frac{d\omega}{dt} = 1.5 - 6t \;\Rightarrow\; 0 = 1.5 - 6t \;\therefore\; t = \frac{1.5}{6} = 0.25$ sec

***Problem* 151.** A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec , it rotates through an angle θ_1 . In the next 2 sec , it rotates through an additional angle θ_2 . The ratio of θ_1/θ_2 is **[AIIMS 1982]**

(a) 1 (b) 2 (c) 3 (d) 5

Solution : (c) From equation of motion $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$\theta_1 = 0 + \frac{1}{2}\alpha(2)^2 = 2\alpha$ .....(i) [As $\omega_1 = 0,\; t = 2\,\text{sec},\; \theta = \theta_1$]

For second condition

$\theta_1 + \theta_2 = 0 + \frac{1}{2}\alpha(4)^2$ [As $\omega_1 = 0,\; t = 2 + 2 = 4$ sec, $\theta = \theta_1 + \theta_2$]

$\theta_1 + \theta_2 = 8\alpha$ ....(ii)

From (i) and (ii) $\theta_1 = 2\alpha,\; \theta_2 = 6\alpha \;\therefore\; \frac{\theta_2}{\theta_1} = 3$

***Problem* 152.** If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2 sec from its start is **[AIIMS 1998]**

(a) 8 *rad* / sec (b) 12 *rad* / sec (c) 24 *rad* / sec (d) 36 *rad* / sec

Solution : (c) $\theta = 2t^3 + 0.5$ and $\omega = \frac{d\theta}{dt} = 6t^2$

at $t = 2$ *sec*, $\omega = 6(2)^2 = 24\; rad/sec$

***Problem* 153.** A grinding wheel attained a velocity of 20 *rad/sec* in 5 sec starting from rest. Find the number of revolutions made by the wheel

(a) $\frac{\pi}{25}$ *rev/ sec* (b) $\frac{1}{\pi}$ *rev/sec* (c) $\frac{25}{\pi}$ *rev/sec* (d) None of these

Solution : (c) $\omega_1 = 0,\; \omega_2 = 20\; rad/sec,\; t = 5\; sec$

$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{20 - 0}{5} = 4\; rad/sec^2$

From the equation $\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(4).(5)^2 = 50\; rad$

$2\pi\, rad$ means 1 revolution. $\therefore$ 50 *Radian* means $\frac{50}{2\pi}$ or $\frac{25}{\pi}$ *rev.*

***Problem* 154.** A grind stone starts from rest and has a constant angular acceleration of 4.0 *rad/sec*². The angular displacement and angular velocity, after 4 *sec.* will respectively be

(a) 32 *rad*, 16 *rad/sec* (b) 16 *rad*, 32 *rad/sec* (c) 64 *rad*, 32 *rad/sec* (d) 32 *rad*, 64 *rad/sec*

Solution : (a) $\omega_1 = 0,\; \alpha = 4\; rad/\text{sec}^2,\; t = 4\; sec$

Angular displacement $\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}4(4)^2 = 32\; rad.$

$\therefore$ Final angular $\omega_2 = \omega_1 + \alpha t = 0 + 4 \times 4 = 16\; rad/sec$

***Problem* 155.** An electric fan is rotating at a speed of 600 *rev/minute*. When the power supply is stopped, it stops after 60 revolutions. The time taken to stop is

(a) 12 *s* (b) 30 *s* (c) 45 *s* (d) 60 *s*

Solution : (a) $\omega_1 = 600 \ rev/\min = 10 \, rev/sec, \ \omega_2 = 0$ and $\theta = 60 \ rev$

From the equation $\omega_2^2 = \omega_1^2 - 2\alpha\theta \Rightarrow 0 = (10)^2 - 2\,\alpha\,60 \ \therefore \alpha = \frac{100}{120} = \frac{5}{6}$

Again $\omega_2 = \omega_1 - \alpha\, t \Rightarrow 0 = \omega_1 - \alpha\, t$

$t = \frac{\omega_1}{\alpha} = \frac{10 \times 6}{5} = 12 \, sec$.

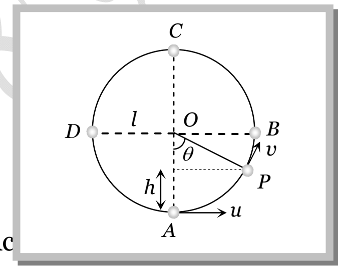
3.23 Motion in Vertical Circle

This is an example of non-uniform circular motion. In this motion body is under the influence of gravity of earth. When body moves from lowest point to highest point. Its speed decrease and becomes minimum at highest point. Total mechanical energy of the body remains conserved and *KE* converts into *PE* and vice versa.

(1) **Velocity at any point on vertical loop :** If *u* is the initial velocity imparted to body at lowest point then. Velocity of body at height *h* is given by

$$v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gl(1 - \cos\theta)} \qquad [\text{As } h = l - l\cos\theta = l\,(1 - \cos\theta)]$$

where *l* in the length of the string

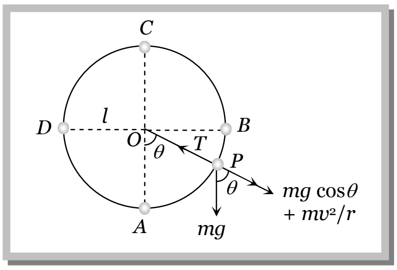


(2) **Tension at any point on vertical loop** : Tension at general point *P*, Ac d law of motion.

Net force towards centre = centripetal force

$$T - mg\cos\theta = \frac{mv^2}{l} \quad \text{or} \quad T = mg\cos\theta + \frac{mv^2}{l}$$

$$T = \frac{m}{l}[u^2 - gl(2 - 3\cos\theta)] \qquad [\text{As } v = \sqrt{u^2 - 2gl(1 - \cos\theta)}\,]$$



(3) **Velocity and tension in a vertical loop at different positions**

Position	Angle	Velocity	Tension
A	0°	u	$\frac{mu^2}{l} + mg$
B	90°	$\sqrt{u^2 - 2gl}$	$\frac{mu^2}{l} - 2mg$
C	180°	$\sqrt{u^2 - 4gl}$	$\frac{mu^2}{l} - 5mg$
D	270°	$\sqrt{u^2 - 2gl}$	$\frac{mu^2}{l} - 2mg$

It is clear from the table that : $T_A > T_B > T_C$ and $T_B = T_D$

$T_A - T_B = 3mg$,

$T_A - T_C = 6mg$

and $T_B - T_C = 3mg$

(4) **Various conditions for vertical motion :**

Velocity at lowest point	Condition
$u_A > \sqrt{5gl}$	Tension in the string will not be zero at any of the point and body will continue the circular motion.
$u_A = \sqrt{5gl}$,	Tension at highest point C will be zero and body will just complete the circle.
$\sqrt{2gl} < u_A < \sqrt{5gl}$,	Particle will not follow circular motion. Tension in string become zero somewhere between points *B* and *C* whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory.
$u_A = \sqrt{2gl}$	Both velocity and tension in the string becomes zero between *A* and *B* and particle will oscillate along semi-circular path.
$u_A < \sqrt{2gl}$	velocity of particle becomes zero between *A* and *B* but tension will not be zero and the particle will oscillate about the point *A*.

Note : ❑ *K.E.* of a body moving in horizontal circle is same throughout the path but the *K.E.* of the body moving in vertical circle is different at different places.

❑ If body of mass *m* is tied to a string of length *l* and is projected with a horizontal velocity *u* then :

Height at which the velocity vanishes is $h = \frac{u^2}{2g}$

Height at which the tension vanishes is $h = \frac{u^2 + gl}{3g}$

(5) **Critical condition for vertical looping :** If the tension at *C* is zero, then body will just complete revolution in the vertical circle. This state of body is known as critical state. The speed of body in critical state is called as critical speed.

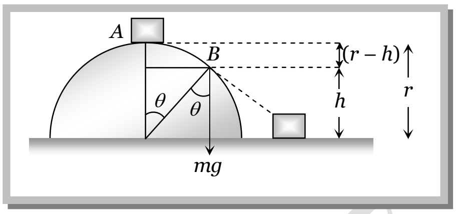
From the above table $T_C = \frac{mu^2}{l} - 5mg = 0 \Rightarrow u = \sqrt{5gl}$

It means to complete the vertical circle the body must be projected with minimum velocity of $\sqrt{5gl}$ at the lowest point.

(6) **Various quantities for a critical condition in a vertical loop at different positions :**

Quantity	Point *A*	Point *B*	Point *C*	Point *D*	Point *P*
Linear velocity (v)	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$	$\sqrt{3gl}$	$\sqrt{gl(3+2\cos\theta)}$
Angular velocity (ω)	$\sqrt{\frac{5g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}(3+2\cos\theta)}$
Tension in String (T)	6 mg	3 mg	0	3 mg	$3mg(1+\cos\theta)$
Kinetic Energy (KE)	$\frac{5}{2}mgl$	$\frac{3}{2}mgl$	$\frac{1}{2}mgl$	$\frac{3}{2}mgl$	$\frac{mgl}{2}(3+2\cos\theta)$
Potential Energy (PE)	0	mgl	2 mgl	mgl	$mgl(1-\cos\theta)$
Total Energy (TE)	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$

(7) **Motion of a block on frictionless hemisphere** : A small block of mass *m* slides down from the top of a frictionless hemisphere of radius *r*. The component of the force of gravity ($mg\cos\theta$) provides required centripetal force but at point *B* it's circular motion ceases and the block lose contact with the surface of the sphere.



For point B, by equating the forces, $mg\cos\theta = \frac{mv^2}{r}$ .....(i)

For point A and B, by law of conservation of energy

Total energy at point A = Total energy at point B

K.E.$_{(A)}$ + P.E.$_{(A)}$ = K.E.$_{(B)}$ + P.E.$_{(B)}$

$$0 + mgr = \frac{1}{2}mv^2 + mgh \Rightarrow v = \sqrt{2g(r-h)} \quad .....(ii)$$

and from the given figure $h = r\cos\theta$ .....(iii)

By substituting the value of v and h from eq^n (ii) and (iii) in eq^n (i)

$$mg\left(\frac{h}{r}\right) = \frac{m}{r}\left(\sqrt{2g(r-h)}\right)^2$$

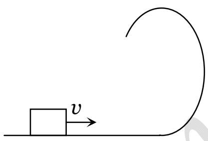
$$\Rightarrow \quad h = 2(r-h) \Rightarrow h = \frac{2}{3}r$$

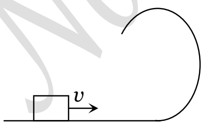
i.e. the block lose contact at the height of $\frac{2}{3}r$ from the ground.

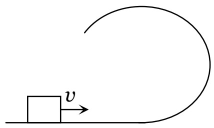
and angle from the vertical can be given by $\cos\theta = \frac{h}{r} = \frac{2}{3}$ $\therefore \theta = \cos^{-1}\frac{2}{3}$.

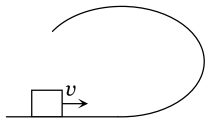
Sample problems based on vertical looping

***Problem* 156.** A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in

(a) 

(b) 

(c) 

(d) 

Solution : (a) Normal reaction at the highest point of the path $R = \frac{mv^2}{r} - mg$

For maximum R, value of the radius of curvature (r) should be minimum and it is minimum in first condition.

***Problem* 157.** A stone tied to string is rotated in a vertical circle. The minimum speed with which the string has to be rotated

[EAMCET (Engg.) 1998; CBSE PMT 1999]

(a) Decreases with increasing mass of the stone (b) Is independent of the mass of the stone

(c) Decreases with increasing in length of the string (d) Is independent of the length of the string

Solution : (b) $v = \sqrt{5gr}$ for lowest point of vertical loop.

$v \propto m^0$ *i.e.* it does not depends on the mass of the body.

***Problem* 158.** A mass m is revolving in a vertical circle at the end of a string of length 20 *cms*. By how much does the tension of the string at the lowest point exceed the tension at the topmost point

(a) 2 mg (b) 4 mg (c) 6 mg (d) 8 mg

Solution : (c) $T_{\text{Lowest point}} - T_{\text{Highest point}} = 6\,mg$ (Always)

Problem 159. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical

(a) $\theta = \cos^{-1}(1/3)$ (b) $\theta = 60^o$ (c) $\theta = \cos^{-1}(2/3)$ (d) $\theta = 0^o$

Solution : (c) Let the string breaks at point *B*.

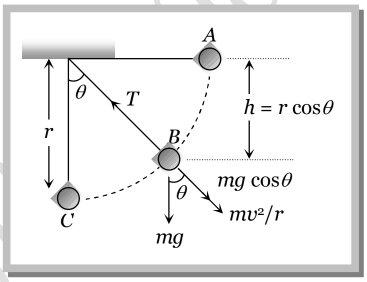
Tension $= mg\cos\theta + \frac{m v_B^2}{r}$ = Breaking strength

$= mg\cos\theta + \frac{m v_B^2}{r} = 2\,mg$ ....(i)

If the bob is released from rest (from point *A*) then velocity acquired by it at point *B*

$v_B = \sqrt{2gh}$

$v_B = \sqrt{2gr\cos\theta}$ ....(ii) [As $h = r\cos\theta$]

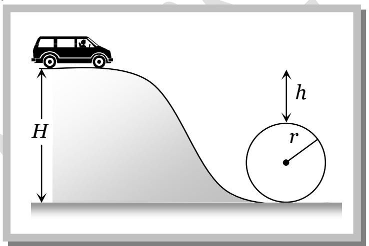


By substituting this value in equation (i)

$mg\cos\theta + \frac{m}{r}(2gr\cos\theta) = 2\,mg$

or $3mg\cos\theta = 2\,mg \Rightarrow \cos\theta = \frac{2}{3}\ \therefore\ \theta = \cos^{-1}\left(\frac{2}{3}\right)$

Problem 160. A toy car rolls down the inclined plane as shown in the fig. It goes around the loop at the bottom. What is the relation between H and h



(a) $\frac{H}{h} = 2$ (b) $\frac{\quad}{h}$ (c) $\frac{\quad}{h}$ (d) $\frac{H}{h} = 5$

Solution : (d) When car rolls down the inclined plane from height *H*, then velocity acquired by it at the lowest point

$v = \sqrt{2gH}$ ....(i)

and for looping of loop, velocity at the lowest point should be $v = \sqrt{5gr}$ ....(ii)

From eq^n (i) and (ii) $v = \sqrt{2gH} = \sqrt{5gr}\ \therefore\ H = \frac{5r}{2}$ ....(iii)

From the figure $H = h + 2r \Rightarrow r = \frac{H-h}{2}$

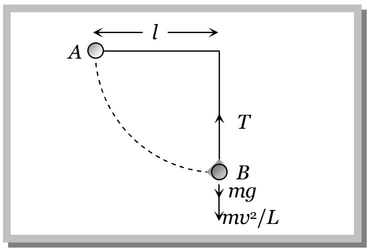
Substituting the value of *r* in equation (iii) we get $H = \frac{5}{2}\left[\frac{H-h}{2}\right] \Rightarrow \frac{H}{h} = 5$

Problem 161. The mass of the bob of a simple pendulum of length L is m. If the bob is left from its horizontal position then the speed of the bob and the tension in the thread in the lowest position of the bob will be respectively.

(a) $\sqrt{2gL}$ and $3mg$ (b) $3mg$ and $\sqrt{2gL}$ (c) $2mg$ and $\sqrt{2gl}$ (d) $2gl$ and $3mg$

Solution : (a) By the conservation of energy

Potential energy at point *A* = Kinetic energy at point *B*



$mg\,l = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{2gl}$

and tension $= mg + \frac{mv^2}{l} \Rightarrow T = mg + \frac{m}{l}(2gl) \Rightarrow T = 3\,mg$

Problem **162.** A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n revolutions per minute. The total tension in the string when the stone is at its lowest point is **[Kerala (Engg.) 2001]**

(a) $m\,\{g + (\pi^2 n^2 r)/900\}$ (b) $m\,(g + \pi\,n r^2)$ (c) $m\,(g + \pi\,nr)$ (d) $m\,(g + n^2 r^2)$

Solution : (a) Tension at lowest point $T = mg + m w^2 r = mg + m 4\pi^2 n^2 r$

If n is revolution per minute then $T = mg + m4\pi^2 \frac{n^2}{3600} r = mg + \frac{m\pi^2 n^2 r}{900} = m\left[g + \frac{\pi^2 n^2 r}{900}\right]$

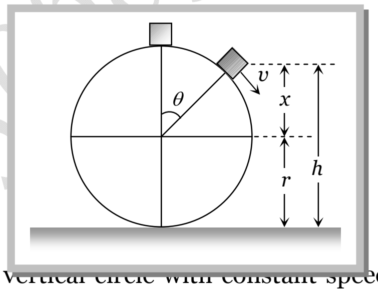
Problem **163.** A particle is kept at rest at the top of a sphere of diameter 42m. When disturbed slightly, it slides down. At what height h from the bottom, the particle will leave the sphere **[IMS-BHU 2003]**

(a) 14 m (b) 28 m (c) 35 m (d) 7 m

Solution : (c) Let the particle leave the sphere at height 'h' from the bottom

We know for given condition $x = \frac{2}{3} r$

and $h = r + x = r + \frac{2}{3} r = \frac{5}{3} r = \frac{5}{3} \times 21 = 35\ m$ [As r= 21 m]

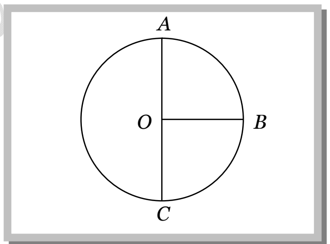


Problem **164.** A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill, when the bucket is at the highest position (Take g = 10 m/sec^2)

(a) 4 m / sec (b) 6.25 m / sec (c) 16m / sec (d) None of these

Solution : (a) $v = \sqrt{g\,r} = \sqrt{10 \times 1.6} = \sqrt{16} = 4\ m/s$

Problem **165.** The ratio of velocities at points A, B and C in vertical circular motion is



(a) 1 : 9 : 25 (b) 1 : 2 : 3 (c) 1 : 3 : 5 (d) $1 : \sqrt{3} : \sqrt{5}$

Solution : (d) $v_A : v_B : v_c = \sqrt{gr} : \sqrt{3\,gr} : \sqrt{5gr} = 1 : \sqrt{3} : \sqrt{5}$

Problem **166.** The minimum speed for a particle at the lowest point of a vertical circle of radius R, to describe the circle is 'v'. If the radius of the circle is reduced to one-fourth its value, the corresponding minimum speed will be

[EAMCET (Engg.) 1999]

(a) $\frac{v}{4}$ (b) $\frac{v}{2}$ (c) 2v (d) 4v

Solution : (b) $v = \sqrt{5gr}$ $\therefore v \propto \sqrt{r}$ So $\frac{v_2}{v_1} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{r/4}{r}} = \frac{1}{2} \Rightarrow v_2 = v/2$

Problem **167.** A body slides down a frictionless track which ends in a circular loop of diameter D, then the minimum height h of the body in term of D so that it may just complete the loop, is **[AIIMS 2000]**

(a) $h = \frac{5D}{2}$ (b) $h = \frac{5D}{4}$ (c) $h = \frac{3D}{4}$ (d) $h = \frac{D}{4}$

Solution : (b) We know $h = \frac{5}{2}r = \frac{5}{2}\left(\frac{D}{2}\right) = \frac{5D}{4}$ [For critical condition of vertical looping]

***Problem* 168.** A can filled with water is revolved in a vertical circle of radius 4*m* and the water just does not fall down. The time period of revolution will be **[CPMT 1985; RPET 1999]**

(a) 1 *sec* (b) 10 *sec* (c) 8 *sec* (d) 4 *sec*

Solution : (d) At highest point $mg = m\omega^2 r \Rightarrow g = \frac{4\pi^2}{T^2}r \Rightarrow 10 = \frac{4\pi^2 4}{T^2} \Rightarrow T^2 = 16 \quad \therefore T = 4\ sec$

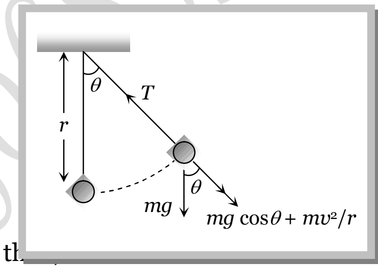
***Problem* 169.** A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from vertical (lowest position) are T_1 and T_2 respectively, Then

(a) $T_1 = T_2$ (b) $T_1 > T_2$ (c) $T_1 < T_2$ (d) $T_1 \geq T_2$

Solution : (b) $T = mg\cos\theta + \frac{mv^2}{r}$

As θ increases T decreases

So $T_1 > T_2$



***Problem* 170.** A mass of $2kg$ is tied to the end of a string of length 1*m*. It is, th … with a constant speed of $5\ ms^{-1}$. Given that $g = 10\ ms^{-2}$. At which of the following locations of tension in the string will be $70\ N$

(a) At the top (b) At the bottom

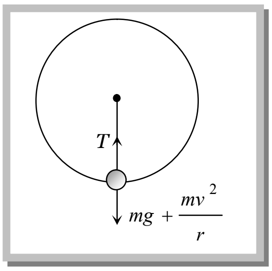
(c) When the string is horizontal (d) At none of the above locations

Solution : (b) Centrifugal force $F = \frac{mv^2}{r} = \frac{2\times(5)^2}{1} = 50\ Newton$

$Weight = mg = 2\times 10 = 20\ Newton$

Tension = 70 *N* (sum of above two forces)

i.e. the mass is at the bottom of the vertical circular path



***Problem* 171.** With what angular velocity should a 20 *m* long cord be rotated such that tension in it, while reaching the highest point, is zero

(a) 0.5 *rad*/*sec* (b) 0.2 *rad*/*sec* (c) 7.5 *rad*/*sec* (d) 0.7 *rad*/*sec*

Solution : (d) $\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{10}{20}} = \sqrt{0.5} = 0.7\ rad/sec$

***Problem* 172.** A body of mass of $100\,g$ is attached to a $1m$ long string and it is revolving in a vertical circle. When the string makes an angle of 60^o with the vertical then its speed is $2\ m/s$. The tension in the string at $\theta = 60^o$ will be

(a) $89\ N$ (b) $0.89\ N$ (c) $8.9\ N$ (d) $0.089\ N$

Solution : (b) $T = mg\cos\theta + \frac{mv^2}{r} = 0.1\times 9.8\times\cos 60 + \frac{0.1\times(2)^2}{1} = 0.49 + 0.4 = 0.89\ Newton$

***Problem* 173.** A body of mass $2kg$ is moving in a vertical circle of radius $2m$. The work done when it moves from the lowest point to the highest point is

(a) 80 J (b) 40 J (c) 20 J (d) 0

Solution : (a) work done = change in potential energy $= 2\,mgr = 2 \times 2 \times 10 \times 2 = 80\ J$

Problem **174.** A body of mass m is tied to one end of a string of length l and revolves vertically in a circular path. At the lowest point of circle, what must be the $K.E.$ of the body so as to complete the circle **[RPMT 1996]**

(a) 5 mgl (b) 4 mgl (c) 2.5 mgl (d) 2 mgl

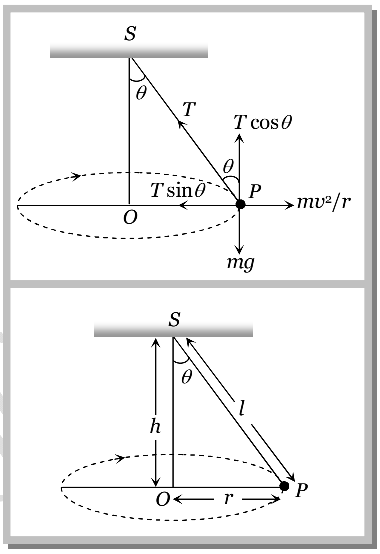
Solution : (c) Minimum velocity at lowest point to complete vertical loop $= \sqrt{5gl}$

So minimum kinetic energy $= \frac{1}{2}m\,(v^2) = \frac{1}{2}m\,(\sqrt{5gl})^2 = \frac{5}{2}mgl = 2.5\,mgl$

3.24 Conical Pendulum

This is the example of uniform circular motion in horizontal plane.

A bob of mass m attached to a light and in-extensible string rotates in a horizontal circle of radius r with constant angular speed ω about the vertical. The string makes angle θ with vertical and appears tracing the surface of a cone. So this arrangement is called conical pendulum.



The force acting on the bob are tension and weight of the bob.

From the figure $T\sin\theta = \dfrac{mv^2}{r}$ ....(i)

and $T\cos\theta = mg$ ....(ii)

(1) Tension in the string : $T = mg\sqrt{1+\left(\dfrac{v^2}{rg}\right)^2}$

$$T = \frac{mg}{\cos\theta} = \frac{mgl}{\sqrt{l^2 - r^2}} \qquad \left[\text{As } \cos\theta = \frac{h}{l} = \frac{\sqrt{l^2 - r^2}}{l}\right]$$

(2) Angle of string from the vertical : $\tan\theta = \dfrac{v^2}{rg}$

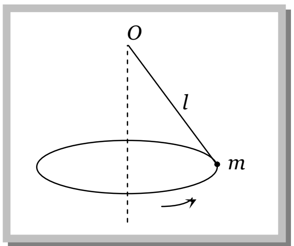
(3) Linear velocity of the bob : $v = \sqrt{gr\tan\theta}$

(4) Angular velocity of the bob : $\omega = \sqrt{\dfrac{g}{r}\tan\theta} = \sqrt{\dfrac{g}{h}} = \sqrt{\dfrac{g}{l\cos\theta}}$

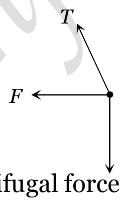
(5) Time period of revolution : $T_P = 2\pi\sqrt{\dfrac{l\cos\theta}{g}} = 2\pi\sqrt{\dfrac{h}{g}} = 2\pi\sqrt{\dfrac{l^2 - r^2}{g}} = 2\pi\sqrt{\dfrac{r}{g\tan\theta}}$

Sample problems based on conical pendulum

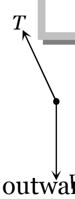
***Problem* 175.** A point mass m is suspended from a light thread of length l, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are



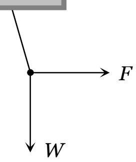
(a)



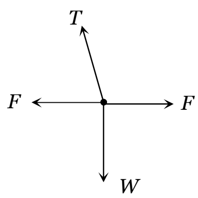
(b)



(c)



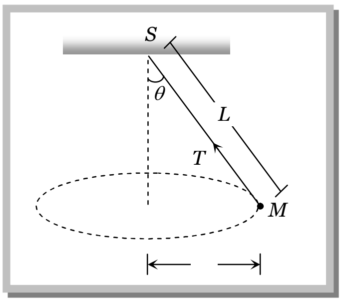
(d)



Solution : (c) Centrifugal force (F) works radially outward,

Weight (w) works downward Tension (T) work along the string and towards the point of suspension

***Problem* 176.** A string of length L is fixed at one end and carries a mass M at the other end. The string makes $2/\pi$ revolutions per *second* around the vertical axis through the fixed end as shown in the figure, then tension in the string is **[BHU 2002]**



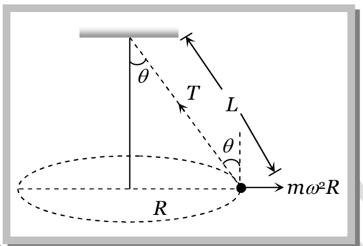
(a) ML (b) $2\,ML$ (c) $4\,ML$ (d) $16\,ML$

Solution : (d) $T\sin\theta = M\omega^2 R$ ..... (i)

$T\sin\theta = M\omega^2 L\sin\theta$ ..... (ii)

From (i) and (ii)

$$T = M\omega^2 L = M4\pi^2 n^2 L = M4\pi^2\left(\frac{2}{\pi}\right)^2 L = 16\,ML$$



Problem **177.** A string of length $1m$ is fixed at one end and a mass of $100\,gm$ is attached at the other end. The string makes $2/\pi$ rev/sec around a vertical axis through the fixed point. The angle of inclination of the string with the vertical is ($g = 10\ m/\sec^2$)

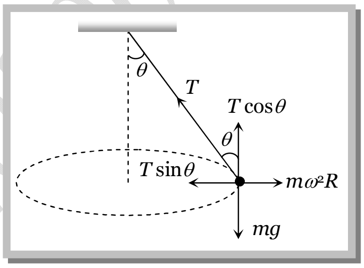
(a) $\tan^{-1}\frac{5}{8}$ (b) $\tan^{-1}\frac{8}{5}$ (c) $\cos^{-1}\frac{8}{5}$ (d) $\cos^{-1}\frac{5}{8}$

Solution : (d) For the critical condition, in equilibrium

$T\sin\theta = m\,\omega^2 r$ and $T\cos\theta = mg$

$$\therefore \tan\theta = \frac{\omega^2 r}{g}$$

$$\Rightarrow \frac{4\pi^2 n^2 r}{g} = \frac{4\pi^2(2/\pi)^2 .1}{10} = \frac{8}{5}$$



*S*ample problems (Miscellaneous)

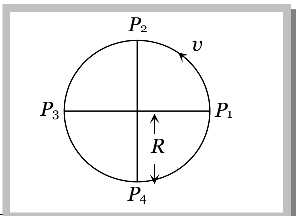
Problem **178.** If the frequency of the rotating platform is *f* and the distance of a boy from the centre is *r*, which is the area swept out per second by line connecting the boy to the centre

(a) πrf (b) $2\pi rf$ (c) $\pi r^2 f$ (d) $2\pi r^2 f$

Solution : (c) Area swept by line in complete revolution $= \pi r^2$

If frequency of rotating platform is *f* per second, then Area swept will be $\pi r^2 f$ per second.

Problem **179.** Figure below shows a body of mass *M* moving with uniform speed *v* along a circle of radius *R*. What is the change in speed in going from P_1 to P_2



(a) Zero (b) $\sqrt{2}v$ (c) $v/\sqrt{2}$ (d) $2v$

Solution : (a) In uniform circular motion speed remain constant. $\therefore$ change in speed is zero.

Problem **180.** In the above problem, what is change in velocity in going from P_1 to P_2

(a) Zero (b) $\sqrt{2}v$ (c) $v/\sqrt{2}$ (d) $2\,v$

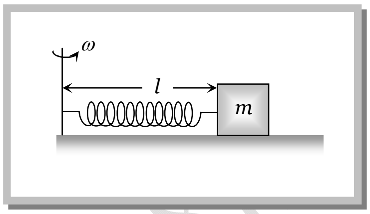
Solution : (b) Change in velocity $= 2v\sin(\theta/2) = 2v\sin\left(\frac{90}{2}\right) = 2v\sin 45 = \frac{2v}{\sqrt{2}} = \sqrt{2}\,v$

Problem **181.** In the above problem, what is the change in angular velocity in going from P_1 to P_2

(a) Zero (b) $\sqrt{2}\,v/R$ (c) $v/\sqrt{2}\,R$ (d) $2v/R$

Solution : (a) Angular velocity remains constant, so change in angular velocity = Zero.

***Problem* 182.** A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length l. The system is rotated about the other end of the spring with an angular velocity ω, in gravity free space. The increase in length of the spring will be



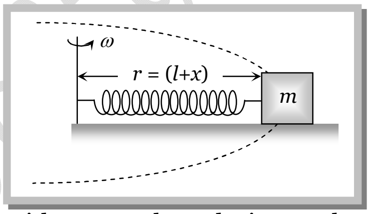
(a) $\frac{m\omega^2 l}{k}$

(b) $\frac{m\omega^2 l}{k - m\omega^2}$

(c) $\frac{m\omega^2 l}{k + m\omega^2}$

(d) None of these

Solution : (b) In the given condition elastic force will provides the required centripetal force



$k\,x = m\,\omega^2 r$

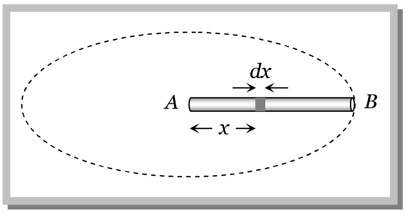
$k\,x = m\,\omega^2 (l + x) \Rightarrow k\,x = m\,\omega^2 l + m\,\omega^2 x \Rightarrow x(k - m\,\omega^2) = m\,\omega^2 l$

$\therefore\ x = \frac{m\,\omega^2 l}{k - m\,\omega^2}$

***Problem* 183.** A uniform rod of mass m and length l rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is

(a) $\frac{1}{2} m\omega^2 x$ (b) $\frac{1}{2} m\omega^2 \frac{x^2}{l}$ (c) $\frac{1}{2} m\omega^2 l\left(1 - \frac{x}{l}\right)$ (d) $\frac{1}{2}\frac{m\omega^2}{l}[l^2 - x^2]$

Solution : (d) Let rod *AB* performs uniform circular motion about point *A*. We have to calculate the tension in the rod at a distance *x* from the axis of rotation. Let mass of the small segment at a distance *x* is *dm*



So $dT = dm\,\omega^2 x = \left(\frac{m}{l}\right) dx\,.\,\omega^2 x = \frac{m\,\omega^2}{l}\,[x\,d\,x]$

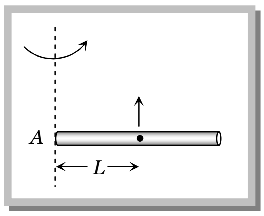
Integrating both sides $\int_x^l dT = \frac{m\,\omega^2}{l}\int_x^l x\,dx \Rightarrow T = \frac{m\,\omega^2}{l}\left[\frac{x^2}{2}\right]_x^l$

$\therefore\ T = \frac{m\,\omega^2}{2l}\left[l^2 - x^2\right]$

***Problem* 184.** A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(a) $\sqrt{\frac{\mu}{\alpha}}$ (b) $\frac{\mu}{\sqrt{\alpha}}$ (c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) Infinitesimal

Solution : (a) Let the bead starts slipping after time *t*



For critical condition

Frictional force provides the centripetal force $m\,\omega^2 L = \mu\,R = \mu\,m \times a_t = \mu m L\,\alpha$

$m\,(\alpha t)^2 L = \mu m L \alpha \Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$ (As $\omega = \alpha t$)

***Problem* 185.** A smooth table is placed horizontally and an ideal spring of spring constant $k = 1000\ N/m$ and unextended length of $0.5m$ has one end fixed to its centre. The other end is attached to a mass of $5kg$ which is moving in a circle with constant speed $20m/s$. Then the tension in the spring and the extension of this spring beyond its normal length are

(a) 500 N, 0.5 m (b) 600 N, 0.6 m (c) 700 N, 0.7 m (d) 800 N, 0.8 m

Solution : (a) $k = 1000$, $m = 5\,kg$, $l = 0.5\,m$, $v = 20\,m/s$ (given)

Restoring force = $kx = \frac{mv^2}{r} = \frac{mv^2}{l+x} \Rightarrow 1000\,x = \frac{5\,(20)^2}{0.5+x} \Rightarrow x = 0.5\,m$

and Tension in the spring = $kx = 1000 \times \frac{1}{2} = 500\ N$

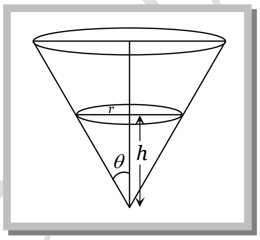
***Problem* 186.** A particle describes a horizontal circle at the mouth of a funnel type vessel as shown in figure. The surface of the funnel is frictionless. The velocity v of the particle in terms of r and θ will be

(a) $v = \sqrt{rg/\tan\theta}$

(b) $v = \sqrt{rg\tan\theta}$

(c) $v = \sqrt{rg\cot\theta}$

(d) $v = \sqrt{rg/\cot\theta}$

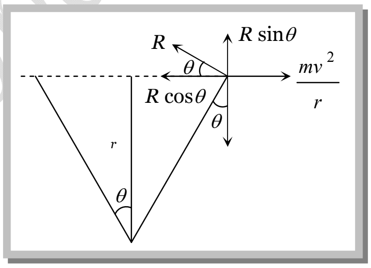


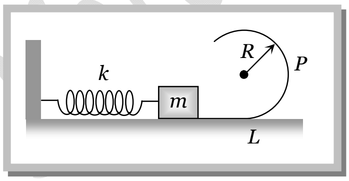
Solution : (c) For uniform circular motion of a particle $\frac{mv^2}{r} = R\cos\theta$ ....(i)

and $mg = R\sin\theta$ ....(ii)

Dividing (i) by (ii)

$\frac{v^2}{rg} = \cot\theta \Rightarrow v = \sqrt{rg\cot\theta}$





***Problem* 187.** Figure shows a smooth track, a part of which is a circle of radius *R*. A block of mass *m* is pushed against a spring constant k fixed at the left end and is then released. Find the initial compression of the spring so that the block presses the track with a force *mg* when it reaches the point *P* [see. Fig], where the radius of the track is horizontal

(a) $\sqrt{\frac{mgR}{3k}}$ (b) $\sqrt{\frac{3gR}{mk}}$ (c) $\sqrt{\frac{3mgR}{k}}$ (d) $\sqrt{\frac{3mg}{kR}}$ *Solution* :

(c) For the given condition, centrifugal force at *P* should be equal to *mg* *i.e.* $\frac{mv_P^2}{R} = mg \therefore v_P = \sqrt{Rg}$

From this we can easily calculate the required velocity at the lowest point of circular track.

$v_p^2 = v_L^2 - 2gR$ (by using formula : $v^2 = u^2 - 2gh$)

$v_L = \sqrt{v_P^2 + 2gR} = \sqrt{Rg + 2gR} = \sqrt{3gR}$

It means the block should possess kinetic energy $= \frac{1}{2}mv_L^2 = \frac{1}{2}m \times 3gR$

And by the law of conservation of energy $\frac{1}{2}kx^2 = \frac{1}{2}3m \times g\,R \Rightarrow x = \sqrt{\frac{3m\,g\,R}{k}}$.