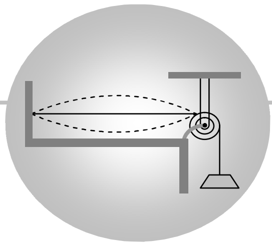


Wave Motion

16.1 Wave

A wave is a disturbance which propagates energy and momentum from one place to the other without the transport of matter.

(1) **Necessary properties of the medium for wave propagation :**

(i) Elasticity : So that particles can return to their mean position, after having been disturbed.

(ii) Inertia : So that particles can store energy and overshoot their mean position.

(iii) Minimum friction amongst the particles of the medium.

(iv) Uniform density of the medium.

(2) **Characteristics of wave motion :**

(i) It is a sort of disturbance which travels through a medium.

(ii) Material medium is essential for the propagation of mechanical waves.

(iii) When a wave motion passes through a medium, particles of the medium only vibrate simple harmonically about their mean position. They do leave their position and move with the disturbance.

(iv) There is a continuous phase difference amongst successive particles of the medium *i.e.*, particle 2 starts vibrating slightly later than particle 1 and so on.

(v) The velocity of the particle during their vibration is different at different position.

(vi) The velocity of wave motion through a particular medium is constant. It depends only on the nature of medium not on the frequency, wavelength or intensity.

(vii) Energy is propagated along with the wave motion without any net transport of the medium.

(3) **Mechanical waves :** The waves which require medium for their propagation are called mechanical waves.

Example : Waves on string and spring, waves on water surface, sound waves, seismic waves.

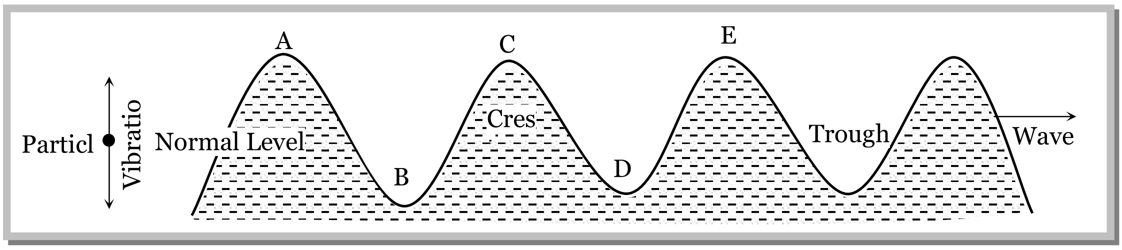
(4) **Non-mechanical waves :** The waves which do not require medium for their propagation are called non- mechanical or electromagnetic waves.

Examples : Light, heat (Infrared), radio waves, γ- rays, X-rays *etc.*

(5) **Transverse waves :** Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.

(i) It travels in the form of crests and troughs.

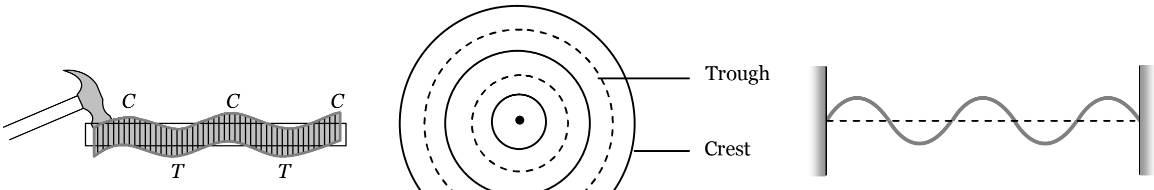
(ii) A crest is a portion of the medium which is raised temporarily above the normal position of rest of the particles of the medium when a transverse wave passes through it.



(iii) A trough is a portion of the medium which is depressed temporarily below the normal position of rest of the particles of the medium, when transverse wave passes through it.

(iv) Examples of transverse wave motion : Movement of string of a sitar or violin, movement of the membrane of a Tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.

(v) Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they can not be transmitted into liquids and gases.

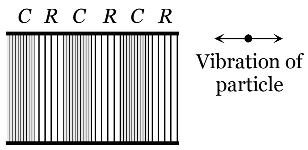


Transverse-wave in a rod

Transverse wave on water surface

Transverse wave in *a* string

(6) **Longitudinal waves :** If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.



(i) It travels in the form of compression and rarefaction.

(ii) A compression (C) is a region of the medium in which particles are compressed.

(iii) A rarefaction (R) is a region of the medium in which particles are rarefied.

(iv) Examples sound waves travel through air in the form of longitudinal waves, Vibration of air column in organ pipes are longitudinal, Vibration of air column above the surface of water in the tube of resonance apparatus are longitudinal.

(v) These waves can be transmitted through solids, liquids and gases because for these waves propagation, volume elasticity is necessary.

(7) **One dimensional wave :** Energy is transferred in a single direction only.

Example : Wave propagating in a stretched string.

(8) **Two dimensional wave :** Energy is transferred in a plane in two mutually perpendicular directions.

Example : Wave propagating on the surface of water.

(9) **Three dimensional wave :** Energy in transferred in space in all direction.

Example : Light and sound waves propagating in space.

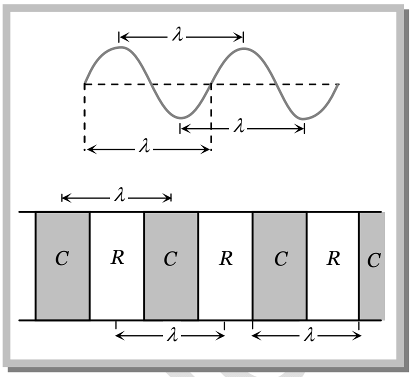
16.2 Important Terms Regarding Wave Motion

(1) **Wavelength :** (i) It is the length of one wave.

(ii) Wavelength is equal to the distance travelled by the wave during the time in which any one particle of the medium completes one vibration about its mean position.

(iii) Wavelength is the distance between any two nearest particles of the medium, vibrating in the same phase.

(iv) Distance travelled by the wave in one time period is known as wavelength.



(v) In transverse wave motion :

λ = Distance between the centres of two consecutive crests.

λ = Distance between the centres of two consecutive troughs.

λ = Distance in which one trough and one crest are contained.

(vi) In longitudinal wave motion :

λ = Distance between the centres of two consecutive compression.

λ = Distance between the centres of two consecutive rarefaction.

λ = Distance in which one compression and one rarefaction contained.

(2) **Frequency :** (i) Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second.

(ii) It is the number of complete wavelengths traversed by the wave in one second.

(iii) Units of frequency are hertz (*Hz*) and per second.

(3) **Time period :** (i) Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

(ii) It is the time taken by the wave to travel a distance equal to one wavelength.

(4) **Relation between frequency and time period :** Time period = 1/Frequency ⇒ $T = 1/n$

(5) **Relation between velocity, frequency and wavelength :** $v = n\lambda$

Velocity (v) of the wave in a given medium depends on the elastic and inertial property of the medium.

Frequency (n) is characterised by the source which produces disturbance. Different sources may produce vibration of different frequencies. Wavelength (λ) will differ to keep $n\lambda = v$ = constant

16.3 Sound Waves

The energy to which the human ears are sensitive is known as sound. In general all types of waves are produced in an elastic material medium, Irrespective of whether these are heard or not are known as sound.

According to their frequencies, waves are divided into three categories :

(1) **Audible or sound waves :** Range 20 *Hz* to 20 *KHz*. These are generated by vibrating bodies such as vocal cords, stretched strings or membrane.

(2) **Infrasonic waves :** Frequency lie below 20 *Hz*.

Example : waves produced during earth quake, ocean waves *etc.*

(3) **Ultrasonic waves :** Frequency greater than 20 *KHz*. Human ear cannot detect these waves, certain creatures such as mosquito, dog and bat show response to these. As velocity of sound in air is 332 *m*/sec so the wavelength of ultrasonics $\lambda < 1.66$ *cm* and for infrasonics $\lambda > 16.6$ *m*.

Note : ❑ **Supersonic speed :** An object moving with a speed greater than the speed of sound is said to move with a supersonic speed.

❑ **Mach number :** It is the ratio of velocity of source to the velocity of sound.

$$\text{Mach Number} = \frac{\text{Velocity of source}}{\text{Velocity of sound}}.$$

❑ Difference between sound and light waves :

(i) For propagation of sound wave material medium is required but no material medium is required for light waves.

(ii) Sound waves are longitudinal but light waves are transverse.

(iii) Wavelength of sound waves ranges from 1.65 *cm* to 16.5 meter and for light it ranges from 4000 Å to 2000 Å.

16.4 Velocity of Sound (Wave motion)

(1) **Speed of transverse wave motion :**

(i) On a stretched string : $v = \sqrt{\frac{T}{m}}$ T = Tension in the string; m = Linear density of string (mass per unit length).

(ii) In a solid body : $v = \sqrt{\frac{\eta}{\rho}}$ η = Modulus of rigidity; ρ = Density of the material.

(2) **Speed of longitudinal wave motion :**

(i) In a solid medium $v = \sqrt{\frac{k + \frac{4}{3}\eta}{\rho}}$ k = Bulk modulus; η = Modulus of rigidity; ρ = Density

When the solid is in the form of long bar $v = \sqrt{\frac{Y}{\rho}}$ Y = Young's modulus of material of rod

(ii) In a liquid medium $v = \sqrt{\frac{k}{\rho}}$

(iii) In gases $v = \sqrt{\frac{k}{\rho}}$

16.5 Velocity of Sound in Elastic Medium

When a sound wave travels through a medium such as air, water or steel, it will set particles of medium into vibration as it passes through it. For this to happen the medium must possess both inertia *i.e.* mass density (so that kinetic energy may be stored) and elasticity (so that PE may be stored). These two properties of matter determine the velocity of sound.

i.e. velocity of sound is the characteristic of the medium in which wave propagate.

$v = \sqrt{\frac{E}{\rho}}$ (E = Elasticity of the medium; ρ = Density of the medium)

Important points

(1) As solids are most elastic while gases least *i.e.* $E_S > E_L > E_G$. So the velocity of sound is maximum in solids and minimum in gases

$$v_{steel} > v_{water} > v_{air}$$

$$5000\ m/s > 1500\ m/s > 330\ m/s$$

As for sound $v_{water} > v_{Air}$ while for light $v_w < v_A$.

Water is rarer than air for sound and denser for light.

The concept of rarer and denser media for a wave is through the velocity of propagation (and not density). Lesser the velocity, denser is said to be the medium and vice-versa.

(2) **Newton's formula :** He assumed that when sound propagates through air temperature remains constant.(*i.e.* the process is isothermal) $v_{air} = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{P}{\rho}}$ As $K = E_\theta = P$; E_θ = Isothermal elasticity; P = Pressure.

By calculation $v_{air} = 279\ m/sec$.

However the experimental value of sound in air is 332 m/sec which is greater than that given by Newton's formula.

(3) **Laplace correction :** He modified Newton's formula assuming that propagation of sound in air as adiabatic process.

$$v = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{E_\phi}{\rho}} \qquad \text{(As } k = E_\phi = \gamma p = \text{Adiabatic elasticity)}$$

$$v = \sqrt{1.41} \times 279 = 331.3\ m/s \quad (\gamma_{\text{Air}} = 1.41)$$

(4) **Effect of density :** $v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v \propto \frac{1}{\sqrt{\rho}}$

(5) **Effect of pressure :** $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$. Velocity of sound is independent of the pressure of gas provided the temperature remains constant. ($P \propto \rho$ when T = constant)

(6) **Effect of temperature :** $v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{T(\text{in } K)}$

When the temperature change is small then $v_t = v_0(1 + \alpha t)$

v_0 = velocity of sound at 0°*C*, v_t = velocity of sound at $t°C$, α = temp-coefficient of velocity of sound.

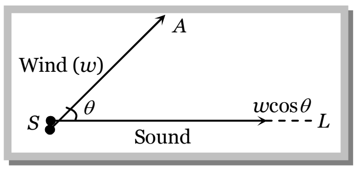
Value of $\alpha = 0.608 \frac{m/s}{^oC} = 0.61$ (Approx.)

Temperature coefficient of velocity of sound is defined as the change in the velocity of sound, when temperature changes by 1°*C*.

(7) **Effect of humidity :** With increase in humidity, density of air decreases. So with rise in humidity velocity of sound increases.

This is why sound travels faster in humid air (rainy season) than in dry air (summer) at the same temperature.

(8) **Effect of wind velocity :** Because wind drifts the medium (air) along its direction of motion therefore the velocity of sound in a particular direction is the algebraic sum of the velocity of sound and the component of wind velocity in that direction. Resultant velocity of sound along $SL = v + w\cos\theta$.



(9) Sound of any frequency or wavelength travels through a given medium with the same velocity.

(v = constant) For a given medium velocity remains constant. All other factors like phase, loudness pitch, quality *etc.* have practically no effect on sound velocity.

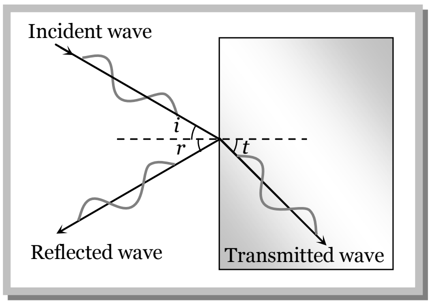
(10) Relation between velocity of sound and root mean square velocity.

$v_{sound} = \sqrt{\frac{\gamma RT}{M}}$ and $v_{rms} = \sqrt{\frac{3RT}{M}}$ so $\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}}$ or $v_{sound} = [\gamma/3]^{1/2} v_{rms}$.

(11) There is no atmosphere on moon, therefore propagation of sound is not possible there. To do conversation on moon, the astronaut uses an instrument which can transmit and detect electromagnetic waves.

16.6 Reflection and Refraction of Waves

When sound waves are incident on a boundary between two media, a part of incident waves returns back into the initial medium (reflection) while the remaining is partly absorbed and partly transmitted into the second medium (refraction) In case of reflection and refraction of sound



(1) The frequency of the wave remains unchanged that means

$\omega_i = \omega_r = \omega_t = \omega$ = constant

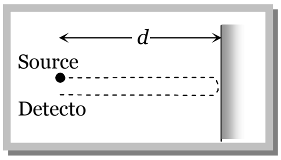
(2) The incident ray, reflected ray, normal and refracted ray all lie in the same plane.

(3) For reflection angle of incidence (i) = Angle of reflection (r)

(4) For refraction $\frac{\sin i}{\sin t} = \frac{v_i}{v_t}$

(5) In reflection from a denser medium or rigid support, phase changes by 180° and direction reverses if incident wave is $y = A_1 \sin(\omega t - kx)$ then reflected wave becomes $y = A_r \sin(\omega t + kx + \pi) = -A_r \sin(\omega t + kx)$.

(6) In reflection from a rarer medium or free end, phase does not change and direction reverses if incident wave is $y = A_I \sin(\omega t - kx)$ then reflected wave becomes $y = A_r \sin(\omega t + kx)$



(7) Echo is an example of reflection.

If there is a sound reflector at a distance d from the source then time interval between original sound and its echo at the site of source will be $t = \frac{2d}{v}$

16.7 Reflection of Mechanical Waves

Medium	Longitudinal wave	Transverse wave	Change in direction	Phase change	Time change	Path change
Reflection from rigid end/denser medium	Compression as rarefaction and vice-versa	Crest as crest and Trough as trough	Reversed	π	$\frac{T}{2}$	$\frac{\lambda}{2}$
Reflection from free end/rarer medium	Compression as compression and rarefaction as rarefaction	Crest as trough and trough as crest	No change	Zero	Zero	Zero

16.8 Progressive Wave

(1) These waves propagate in the forward direction of medium with a finite velocity.

(2) Energy and momentum are transmitted in the direction of propagation of waves without actual transmission of matter.

(3) In progressive waves, equal changes in pressure and density occurs at all points of medium.

(4) Various forms of progressive wave function.

(i) $y = A \sin(\omega t - kx)$ where y = displacement
A = amplitude
ω = angular frequency
n = frequency
k = propagation constant
T = time period

(ii) $y = A \sin (\omega t - \frac{2\pi}{\lambda} x)$

(iii) $y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$

(iv) $y = A \sin \frac{2\pi}{\lambda}(vt - x)$

(v) $y = A \sin \omega \left(t - \frac{x}{v}\right)$

Important points

(a) If the sign between *t* and *x* terms is negative the wave is propagating along positive *X*-axis and if the sign is positive then the wave moves in negative *X*-axis direction.

(b) The coefficient of sin or cos functions *i.e.* Argument of sin or cos function *i.e.* $(\omega t - kx)$ = Phase.

(c) The coefficient of *t* gives angular frequency $\omega = 2\pi n = \frac{2\pi}{T} = vk$.

(d) The coefficient of *x* gives propagation constant or wave number $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$.

(e) The ratio of coefficient of *t* to that of *x* gives wave or phase velocity. *i.e.* $v = \frac{\omega}{k}$.

(f) When a given wave passes from one medium to another its frequency does not change.

(g) From $v = n\lambda \Rightarrow v \propto \lambda \; \Theta \; n = \text{constant} \Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$.

(5) **Some terms related to progressive waves**

(i) **Wave number ($\bar{n}$) :** The number of waves present in unit length is defined as the wave number ($\bar{n}$) = $\frac{1}{\lambda}$.

Unit = $meter^{-1}$; Dimension = $[L^{-1}]$.

(ii) **Propagation constant (*k*) :** $k = \frac{\phi}{x} = \frac{\text{Phase difference between particles}}{\text{Distance between them}}$

$$k = \frac{\omega}{v} = \frac{\text{Angular velocity}}{\text{Wave velocity}} \quad \text{and} \quad k = \frac{2\pi}{\lambda} = 2\pi\bar{\lambda}$$

(iii) **Wave velocity (*v*) :** The velocity with which the crests and troughs or compression and rarefaction travel in a medium, is defined as wave velocity $v = \frac{\omega}{k} = n\lambda = \frac{\omega\lambda}{2\pi} = \frac{\lambda}{T}$.

(iv) **Phase and phase difference :** Phase of the wave is given by the argument of sine or cosine in the equation of wave. It is represented by $\phi(x, t) = \frac{2\pi}{\lambda}(vt - x)$.

(v) At a given position (for fixed value of *x*) phase changes with time (*t*).

$$\frac{d\phi}{dt} = \frac{2\pi v}{\lambda} = \frac{2\pi}{T} \Rightarrow d\phi = \frac{2\pi}{T}.dt \Rightarrow \text{Phase difference} = \frac{2\pi}{T} \times \text{Time difference.}$$

(vi) At a given time (for fixed value of *t*) phase changes with position (*x*).

$\frac{d\phi}{dx} = \frac{2\pi}{\lambda} \Rightarrow d\phi = \frac{2\pi}{\lambda} \times dx \Rightarrow$ Phase difference = $\frac{2\pi}{\lambda} \times$ Path difference

$\Rightarrow$ Time difference = $\frac{T}{\lambda} \times$ Path difference

Sample problems based on Progressive wave

***Problem* 1.** The speed of a wave in a certain medium is 960 *m/sec*. If 3600 waves pass over a certain point of the medium in 1 minute, the wavelength is **[MP PMT 2000]**

(a) 2 *meters* (b) 4 *meters* (c) 8 *meters* (d) 16 *meters*

Solution : (d) $v = 960\ m/s$; $n = \frac{3600}{60} Hz$. So $\lambda = \frac{v}{n} = \frac{960}{60} = 16$ *meters*.

***Problem* 2.** A simple harmonic progressive wave is represented by the equation $y = 8 \sin 2\pi (0.1x - 2t)$ where x and y are in *cm* and t is in seconds. At any instant the Phase difference between two particles separated by 2.0 *cm* in the x-direction is **[MP PMT 2000]**

(a) 18° (b) 36° (c) 54° (d) 72°

Solution : (d) $y = 8 \sin 2\pi \left(\frac{x}{10} - 2t\right)$ given by comparing with standard equation $y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$\lambda = 10\ cm$

So Phase Difference = $\frac{2\pi}{\lambda} \times$ path difference = $\frac{2\pi}{10} \times 2 = \frac{2}{5} \times 180° = 72°$

***Problem* 3.** The frequency of sound wave is n and its velocity is v if the frequency is increased to $4n$ the velocity of the wave will be **[MP PET 2000]**

(a) v (b) $2v$ (c) $4v$ (d) $v/4$

Solution : (a) Wave velocity does not depends on the frequency. It depends upon the Elasticity and inertia of the medium.

***Problem* 4.** The displacement of a particle is given by $x = 3 \sin (5\pi t) + 4 \cos (5\pi t)$ The amplitude of particle is

[MP PMT 1999]

(a) 3 (b) 4 (c) 5 (d) 7

Solution : (c) Standard equation : $x = a \sin \omega t + b \cos \omega t$

$$x = \sqrt{a^2 + b^2} \sin\left(\omega t + \tan^{-1}(b/a)\right)$$

Given equation $x = 3 \sin(5\pi t) + 4 \cos(5\pi t)$

$$x = \sqrt{9 + 16} \sin\left(5\pi t + \tan^{-1} 4/3\right)$$

$$x = 5 \sin(5\pi t + \tan^{-1}(4/3))$$

***Problem* 5.** The equation of a transverse wave travelling on a rope is given by $y = 10 \sin \pi (0.01 x - 2.00 t)$ where y and x are in *cm* and t in seconds. The maximum transverse speed of a particle in the rope is about **[MP PET 1999]**

(a) 63 *cm/sec* (b) 75 *cm/s* (c) 100 *cm/sec* (d) 121 *cm/sec*

Solution : (a) Standard eq. of travelling wave $y = A \sin (kx - \omega t)$

By comparing with the given equation $y = 10 \sin (0.01\pi x - 2\pi t)$

$A = 10\ cm$, $\omega = 2\pi$

Maximum particle velocity = $A\omega = 2\pi \times 10 = 63$ *cm/sec*

***Problem* 6.** In a wave motion $y = a \sin (kx - \omega t)$, y can represents

(a) Electric Field (b) magnetic field (c) Displacement (d) Pressure

Solution : (a,b,c,d)

***Problem* 7.** Find the ratio of the speed of sound in nitrogen gas to that of helium gas, at 300 *k* is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\sqrt{\frac{3}{5}}$ (d) $\frac{4}{5}$

Solution : (c) $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v_N}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}} \frac{M_{He}}{M_{N2}}} = \sqrt{\frac{7/5}{5/3} \cdot \frac{4}{28}} = \sqrt{\frac{3}{5}}.$$

***Problem* 8.** The displacement *x* (in metres) of a particle performing simple harmonic motion is related to time *t* (in seconds) as $x = 0.05 \cos\left(4\pi t + \frac{\pi}{4}\right)$. The frequency of the motion will be **[MP PMT / PET 1998]**

(a) 0.5 *Hz* (b) 1.0 *Hz* (c) 1.5 *Hz* (d) 2.0 *Hz*

Solution : (d) From the given equation, coefficient of $t = \omega = 4\pi$

$$\therefore \quad n = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = 2\,Hz$$

***Problem* 9.** A wave is represented by the equation $Y = 7 \sin\left(7\pi t - 0.04\pi x + \frac{\pi}{3}\right)$ *x* is in meters and *t* is in seconds. The speed of the wave is **[MP PET 1996]**

(a) 175 *m/sec* (b) 49 π *m/s* (c) $\frac{49}{\pi} m/s$ (d) 0.28 $\pi\, m/s$

Solution : (a) Standard equation $y = A \sin(\omega t - kx + \phi_0)$

In a given equation $\omega = 7\pi, k = 0.04\pi$

$$v = \frac{\omega}{k} = \frac{7\pi}{.04\pi} = 175\ m/sec$$

***Problem* 10.** A wave is represented by the equation y = 0.5 sin (10 t + x)*m*. It is a travelling wave propagating along the *x* direction with velocity. **[Roorkee 1995]**

(a) 10 *m/s* (b) 20 *m/s* (c) 5 *m/s* (d) None of these

Solution : (a) $v = \omega / k = 10/1 = 10\ m/s$

***Problem* 11.** A transverse progressive wave on a stretched string has a velocity of 10 ms^{-1} and a frequency of 100 *Hz*. The phase difference between two particles of the string which are 2.5 *cm* apart will be

(a) $\pi/8$ (b) $\pi/4$ (c) $3\pi/8$ (d) $\pi/2$

Solution : (d) $\lambda = v/n = \frac{10}{100} = 0.1\,m = 10\,cm$

Phase difference = $\frac{2\pi}{\lambda}$ × path difference $= \frac{2\pi}{10} \times 2.5 = \frac{\pi}{2}$

***Problem* 12.** In a stationary wave, all particles are **[MP PMT 1994]**

(a) At rest at the same time twice in every period of oscillation
(b) At rest at the same time only once in every period of oscillation
(c) Never at rest at the same time
(d) Never at rest at all

Solution : (a)

***Problem* 13.** The path difference between the two waves

$y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and $y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$ is **[MP PMT 1994]**

(a) $\frac{\lambda}{2\pi}\phi$ (b) $\frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$ (c) $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$ (d) $\frac{2\pi}{\lambda}(\phi)$

Solution : (b) $y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$; $y_2 = a_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$

Phase difference $= \left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right) - \left(\omega t - \frac{2\pi x}{\lambda}\right) = \left(\phi + \frac{\pi}{2}\right)$

Path difference $= \frac{\lambda}{2\pi} \times$ Phase difference $= \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

***Problem* 14.** A plane wave is described by the equation $y = 3\cos\left(\frac{x}{4} - 10t - \frac{\pi}{2}\right)$. The maximum velocity of the particles of the medium due to this wave is

(a) 30 (b) $3\pi/2$ (c) 3/4 (d) 40

Solution : (a) Maximum velocity = $A\omega = 3 \times 10 = 30$

***Problem* 15.** A wave represented by the given equation $y = A \sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$ where x is in meter and t is in second. The expression represents

(a) A wave travelling in the positive x-direction with a velocity of 1.5 *m/sec*
(b) A wave travelling in the negative x-direction with a velocity of 1.5 *m/sec*
(c) A wave travelling in the negative x-direction with a wavelength of 0.2 *m*
(d) A wave travelling in the positive x-direction with a wavelength of 0.2 *m*

Solution : (b, c) By comparing with standard equation $Y = A\sin(kx + \omega t + \pi/3)$

$K = 10\pi$, $\omega = 15\pi$

We know that : $v = \frac{\omega}{k} = 1.5$ *m/sec*; $\lambda = \frac{2\pi}{k} = 0.2$ *meter*.

***Problem* 16.** A transverse wave is described by the equation $Y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$ The maximum particle velocity is four times the wave velocity if

(a) $\lambda = \frac{\pi y_0}{4}$ (b) $\lambda = \frac{\pi y_0}{2}$ (c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$

Solution : (b) Maximum particle velocity = 4 wave velocity

$A\omega = 4f\lambda$

$y_0 2\pi f = 4f\lambda$

$\lambda = \frac{\pi y_0}{2}$

***Problem* 17.** The equation of a wave travelling in a string can be written as $y = 3\cos\pi(100t - x)$ Its wavelength is **[MP PMT 1991, 94, 97; MNR 1985]**

(a) 100 *cm* (b) 2 *cm* (c) 5 *cm* (d) None of these

Solution : (b) $y = A\cos(\omega t - kx)$ – standard equation

$y = 3\cos(100\pi t - \pi x)$ – given equation

So $K = \pi$ and $\lambda = \frac{2\pi}{k} = 2$ *cm*

***Problem* 18.** A plane wave is represented by $x = 1.2\sin(314t + 12.56y)$ where x and y are distances measured along in x and y direction in meter and t is time in seconds. This wave has **[MP PET 1991]**

(a) A wave length of 0.25 *m* and travels m + *ve* x-direction
(b) A wavelength of 0.25 *m* and travels in + *ve* y-direction
(c) A wavelength of 0.5 *m* and travels in – *ve* y-direction

(d) A wavelength of 0.5 m and travels in – ve x-direction

Solution : (c) From given equation $k = 12.56$

$\lambda = \frac{2\pi}{k} 0.5\, m$ direction = – y

***Problem* 19.** A wave is reflected from a rigid support. The change in phase on reflection will be

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) 2π

Solution : (c)

***Problem* 20.** The equation of displacement of two waves are given as $y_1 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$; $y_2 = 5 [\sin 3\pi t + \sqrt{3} \cos 3\pi t]$

Then what is the ratio of their amplitudes **[AIIMS 1997]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) None of these

Solution : (c) $y_2 = 5 [\sin 3\pi t + \sqrt{3} \cos 3\pi t] = 5\sqrt{1+3} \sin\left(3\pi t + \frac{\pi}{3}\right) = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$

So, $A_1 = 10$ and $A_2 = 10$

***Problem* 21.** The equation of a wave travelling on a string is $y = 4 \sin \frac{\pi}{2}\left(8t - \frac{x}{8}\right)$ if x and y are in cm, then velocity of wave is **[MP PET 1990]**

(a) 64 cm/sec in – x direction (b) 32 cm/sec in – x direction

(c) 32 cm/sec in +x direction (d) 64 cm/sec in +x direction

Solution : (d) $y = 4 \sin\left(4\pi t - \frac{\pi}{16}.x\right)$

$\omega = 4\pi, k = \pi/16$

$v = \frac{\omega}{k} = \frac{4\pi}{\pi/16} = 64\ cm/\sec$ in + x direction.

***Problem* 22.** The equation of wave is $y = 2 \sin \pi(0.5x - 200t)$ where x and y are expressed in cm and t in sec. The wave velocity is **[MP PMT 1986]**

(a) 100 cm/sec (b) 200 cm/sec (c) 300 cm/sec (d) 400 cm/sec

Solution : (d) $v = \frac{\omega}{k} = \frac{200\pi}{0.5\pi} = 400\ cm/sec$

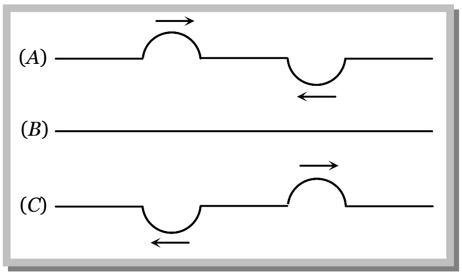
16.9 Principle of Superposition

The displacement at any time due to any number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due each one of the waves at that point at the same time.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3$ .... are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement. $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$ ...

Examples

(i) Radio waves from different stations having different frequencies cross the antenna. But our T.V/Radio set can pick up any desired frequency.



(ii) When two pulses of equal amplitude on a string approach each other [fig. (A)], then on meeting, they superimpose to produce a resultant pulse of zero amplitude [fig (B)]. After crossing, the two pulses travel independently as shown in [fig (C)] as if nothing had happened.

Important applications of superposition principle :

(a) Interference of waves (b) Stationary waves (c) Beats.

16.10 Interference of Sound Waves

When two waves of same frequency, same wavelength, same velocity (nearly equal amplitude) moves in the same direction, Their superimposition results in the interference. Due to interference the resultant intensity of sound at that point is different from the sum of intensities due to each wave separately. This modification of intensity due to superposition of two or more waves is called interference.

Let at a given point two waves arrives with phase difference ϕ and the equation of these waves is given by

$y_1 = a_1 \sin \omega t$, $y_2 = a_2 \sin(\omega t + \phi)$ then by the principle of superposition

$\vec{y} = \vec{y}_1 + \vec{y}_2 \Rightarrow y = A \sin(\omega t + \theta)$ where $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$ and $\tan\theta = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$

and since Intensity $\propto A^2$.

So $I = a_1^2 + a_2^2 + 2a_1, a_2 \cos\phi \Rightarrow I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Important points

(1) **Constructive interference :** Intensity will be maximum

when $\phi = 0, 2\pi, 4\pi, ... 2\pi n$; where n = 0,1,2 ....

when x = 0, λ, 2λ, ... $n\lambda$; where n = 0, 1 ....

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \propto (A_1 + A_2)^2$

It means the intensity will be maximum at those points where path difference is an integral multiple of wavelength λ. These points are called points of constructive interference or interference maxima.

(2) **Destructive interference :** Intensity will be minimum

when $\phi = \pi, 3\pi, 5\pi ...... (2n-1)\pi$; where n= 1, 2, 3 ....

when x = λ/2, 3λ/2, ... $(2n-1)\lambda/2$; where n = 1, 2, 3........

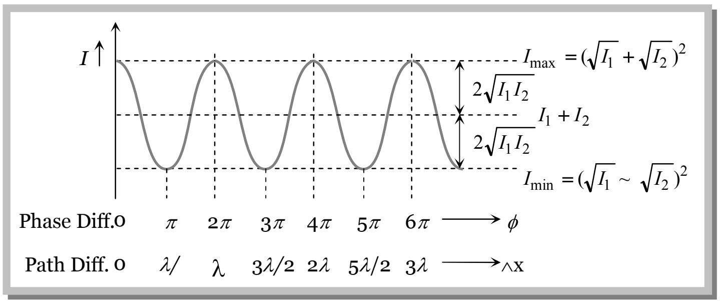
$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \Rightarrow I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \propto (A_1 \sim A_2)^2$

(3) All maxima are equally spaced and equally loud. Same is also true for minima. Also interference maxima and minima are alternate as for maximum $\Delta x = 0, \lambda, 2\lambda ....etc.$ and for minimum $\Delta x = \dfrac{\lambda}{2}, \dfrac{3\lambda}{2}, \dfrac{5\lambda}{2} .... etc$.

(4) $\dfrac{I_{max}}{I_{min}} = \dfrac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2} = \dfrac{(A_1 + A_2)^2}{(A_1 \sim A_2)^2}$ with $\dfrac{I_1}{I_2} = \dfrac{A_1^2}{A_2^2}$

(5) If $I_1 = I_2 = I_0$ then $I_{max} = 4I_o$ and I_{min} = 0

(6) In interference the intensity in maximum $\left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ exceeds the sum of individual



intensities ($I_1 + I_2$) by an amount $2\sqrt{I_1 I_2}$ while in minima $\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$ lacks $(I_1 + I_2)$ by the same amount $2\sqrt{I_1 I_2}$.

Hence in interference energy is neither created nor destroyed but is redistributed.

16.11 Standing Waves or Stationary Waves

When two sets of progressive wave trains of same type (both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

Characteristics of standing waves :

(1) The disturbance confined to a particular region between the starting point and reflecting point of the wave.

(2) There is no forward motion of the disturbance from one particle to the adjoining particle and so on, beyond this particular region.

(3) The total energy associated with a stationary wave is twice the energy of each of incident and reflected wave. But there is no flow or transference of energy along the stationary wave.

(4) There are certain points in the medium in a standing wave, which are permanently at rest. These are called nodes. The distance between two consecutive nodes is $\frac{\lambda}{2}$

(5) Points of maximum amplitude is known as antinodes. The distance between two consecutive antinodes is also $\lambda / 2$. The distance between a node and adjoining antinode is $\lambda / 4$.

(6)The medium splits up into a number of segments. Each segment is vibrating up and down as a whole.

(7) All the particles in one particular segment vibrate in the same phase. Particles in two consecutive segments differ in phase by 180°.

(8) All the particles except those at nodes, execute simple harmonic motion about their mean position with the same time period.

(9) The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.

(10) Twice during each vibration, all the particles of the medium pass simultaneously through their mean position.

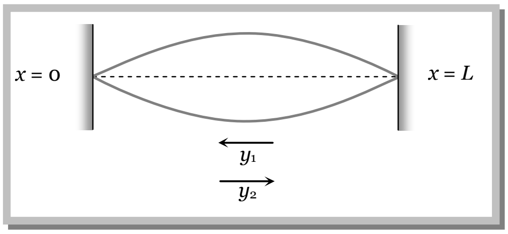
(11) The wavelength and time period of stationary waves are the same as for the component waves.

(12) Velocity of particles while crossing mean position varies from maximum at antinodes to zero at nodes.

(13) In standing waves, if amplitude of component waves are not equal. Resultant amplitude at nodes will be minimum (but not zero). Therefore, some energy will pass across nodes and waves will be partially standing.

16.12 Standing Waves on a String

When a string under tension is set into vibration, transverse harmonic waves propagate along its length. When the length of string is fixed, reflected waves will also exist. The incident and reflected waves will superimpose to produce transverse stationary waves in a string

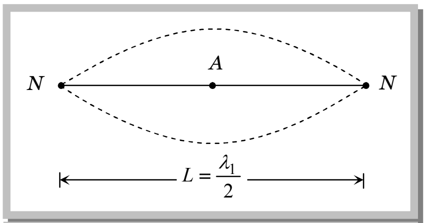


Incident wave $y_1 = a \sin \frac{2\pi}{\lambda}(vt + x)$

Reflected wave $y_2 = a \sin \frac{2\pi}{\lambda}\left[(vt - x) + \pi\right] = -a \sin \frac{2\pi}{\lambda}(vt - x)$

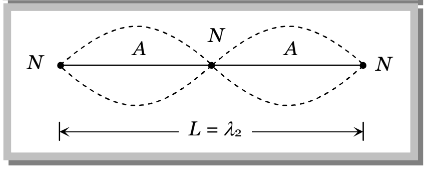
According to superposition principle : $y = y_1 + y_2 = 2\, a \cos \frac{2\pi vt}{\lambda} \sin \frac{2\pi x}{\lambda}$

General formula for wavelength $\lambda = \frac{2L}{n}$ where n = 1,2,3, ... correspond to 1st , 2nd, 3rd modes of vibration of the string.



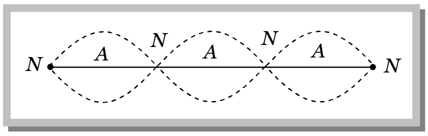
(1) First normal mode of vibration $n_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \Rightarrow n_1 = \frac{1}{2L}\sqrt{\frac{T}{m}}$

This mode of vibration is called the fundamental mode and the frequency is called fundamental frequency. The sound from the note so produced is called fundamental note or first harmonic.



(2) Second normal mode of vibration : $n_2 = \frac{v}{\lambda_2} = \frac{v}{L} = \frac{2v}{2L} = 2(n_1)$

This is second harmonic or first over tone.



(3) Third normal mode of vibration : $n_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3n_1$

This is third harmonic or second overtone.

Position of nodes : $x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n} ....\,..L$

For first mode of vibration $x = 0, x = L$ [Two nodes]

For second mode of vibration $x = 0, x = \frac{L}{2}, x = L$ [Three nodes]

For third mode of vibration $x = 0, x = \frac{L}{3}, x = \frac{2L}{3}, x = L$ [Four nodes]

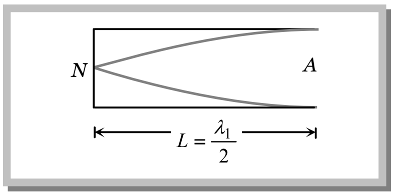
Position of antinodes : $x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} .... \frac{(2\eta - 1)L}{2n}$

For first mode of vibration $x = L/2$ [One antinode]

For second mode of vibration x = $\frac{L}{4}, \frac{3L}{4}$ [Two antinode]

16.13 Standing Wave in a Closed Organ Pipe

Organ pipes are the musical instrument which are used for producing musical sound by blowing air into the pipe. Longitudinal stationary waves are formed on account of superimposition of incident and reflected longitudinal waves.

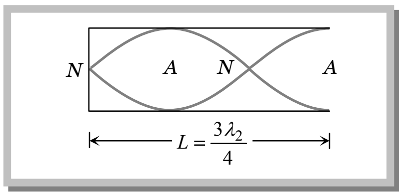


Equation of standing wave y = $2a \cos \frac{2\pi vt}{\lambda} \sin \frac{2\pi x}{\lambda}$

General formula for wavelength $\lambda = \frac{4L}{(2n-1)}$

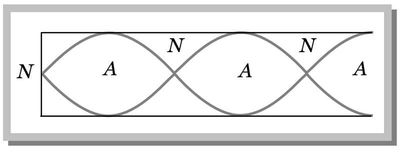
(1) First normal mode of vibration : $n_1 = \frac{v}{4L}$

This is called fundamental frequency. The note so produced is called fundamental note or first harmonic.



(2) Second normal mode of vibration : $n_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3\,n_1$

This is called *third harmonic* or *first overtone*.



(3) Third normal mode of vibration : $n_3 = \frac{5v}{4L} = 5n_1$

This is called *fifth harmonic* or *second overtone*.

Position of nodes : $x = 0,\ \frac{2L}{(2n-1)}, \frac{4L}{(2n-1)}, \frac{6L}{(2n-1)} \ldots\ldots\ldots \frac{2nL}{(2n-1)}$

For first mode of vibration $x = 0$ [One node]

For second mode of vibration $x = 0, x = \frac{2L}{3}$ [Two nodes]

For third mode of vibration $x = 0,\ x = \frac{2L}{5}, \frac{4L}{5}$ [Three nodes]

Position of antinode : $x = \frac{L}{2n-1}, \frac{3L}{2n-1}, \frac{5L}{2n-1} \ldots\ldots\ldots, L$

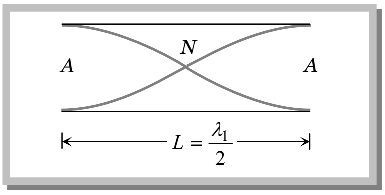
For first mode of vibration $x = L$ [One antinode]

For second mode of vibration $x = \frac{L}{3}, x = L$ [Two antinode]

For third mode of vibration $x = \frac{L}{5}, \frac{3L}{5}, L$ [Three antinode]

16.14 Standing Waves in Open Organ Pipes

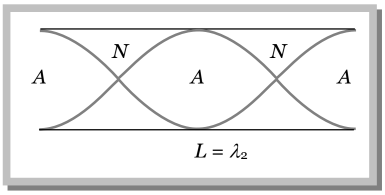
General formula for wavelength



$\lambda = \frac{2L}{n}$ where n = 1,2,3 ....

(1) First normal mode of vibration : $n_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

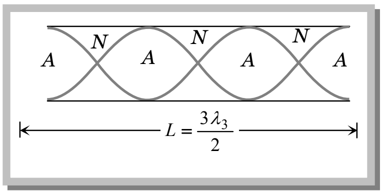
This is called fundamental frequency and the note so produced is called *fundamental note* or *first harmonic*.



(2) Second normal mode of vibration

$n_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2n_1 \Rightarrow n_2 = 2n_1$

This is called *second harmonic* or *first overtone*.



(3) Third normal mode of vibration $n_3 = \frac{v}{\lambda_3} = \frac{3v}{2L},\ n_3 = 3n_1$

This is called *third harmonic* or *second overtone*.

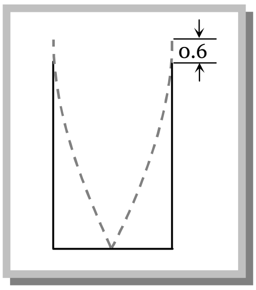
Important points

(i) Comparison of closed and open organ pipes shows that fundamental note in open organ pipe $\left(n_1 = \frac{v}{2L}\right)$ has double the frequency of the fundamental note in closed organ pipe $\left(n_1 = \frac{v}{4L}\right)$.

Further in an open organ pipe all harmonics are present whereas in a closed organ pipe, only alternate harmonics of frequencies $n_1, 3n_1, 5n_1, ...$ *etc* are present. The harmonics of frequencies $2n_1$, $4n_1$, $6n_1$ ...... are missing.

Hence musical sound produced by an open organ pipe is sweeter than that produced by a closed organ pipe.

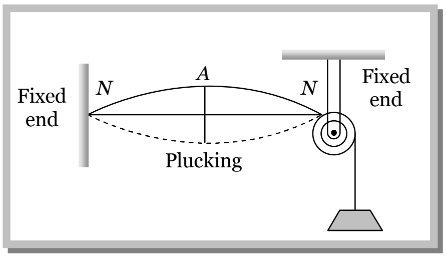
(ii) Harmonics are the notes/sounds of frequency equal to or an integral multiple of fundamental frequency (n). Thus the first, second, third, harmonics have frequencies $n_1, 2n_1, 3n_1$, ...



(iii) Overtones are the notes/sounds of frequency twice/thrice/ four times the fundamental frequency (n) *eg*. $2n, 3n, 4n$ .... and so on.

(iv) In organ pipe an antinode is not formed exactly at the open end rather it is formed a little distance away from the open end outside it. The distance of antinode from the open end of the pipe is known as end correction.

16.15 Vibration of a String



Fundamental frequency $n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

General formula $n_p = \frac{p}{2L}\sqrt{\frac{T}{m}}$

L = Length of string, T = Tension in the string

m = Mass per unit length (linear density), p = mode of vibration

Important points

(1) As a string has many natural frequencies, so when it is excited with a tuning fork, the string will be in resonance with the given body if any of its natural frequencies concides with the body.

(2) (i) $n \propto \frac{1}{L}$ if T and m are constant (ii) $n \propto \sqrt{T}$ if L and m are constant (iii) $n \propto \frac{1}{\sqrt{m}}$ if T and L are constant

(3) If M is the mass of the string of length L, $m = \frac{M}{L}$

So $n = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{T}{M/L}} = \frac{1}{2}\sqrt{\frac{T}{ML}} = \frac{1}{2L}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{2Lr}\sqrt{\frac{T}{\pi\rho}}$ where $m = \pi r^2 \rho$ (r = Radius, ρ = Density)

16.16 Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S. No.	Parameter	Stretched string	Open organ pipe	Closed organ pipe
(1)	Fundamental frequency or 1st harmonic	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$

(2)	Frequency of 1st overtone or 2nd harmonic	$n_2 = 2n_1$	$n_2 = 2n_1$	Missing
(3)	Frequency of 2nd overtone or 3rd harmonic	$n_3 = 3n_1$	$n_3 = 3n_1$	$n_3 = 3n_1$
(4)	Frequency ratio of overtones	2 : 3 : 4...	2 : 3 : 4...	3 : 5 : 7...
(5)	Frequency ratio of harmonics	1 : 2 : 3 : 4...	1 : 2 : 3 : 4...	1 : 3 : 5 : 7...
(6)	Nature of waves	Transverse stationary	Longitudinal stationary	Longitudinal stationary

16.17 Beats

When two sound waves of slightly different frequencies, travelling in a medium along the same direction, superimpose on each other, the intensity of the resultant sound at a particular position rises and falls regularly with time. This phenomenon of regular variation in intensity of sound with time at a particular position is called beats.

Important points

(1) **One beat :** If the intensity of sound is maximum at time t = 0, one beat is said to be formed when intensity becomes maximum again after becoming minimum once in between.

(2) **Beat period :** The time interval between two successive beats (*i.e.* two successive maxima of sound) is called beat period.

(3) **Beat frequency :** The number of beats produced per second is called beat frequency.

(4) **Persistence of hearing :** The impression of sound heard by our ears persist in our mind for 1/10th of a second. If another sound is heard before 1/10 second is over, the impression of the two sound mix up and our mind cannot distinguish between the two.

So for the formation of distinct beats, frequencies of two sources of sound should be nearly equal (difference of frequencies less than 10)

(5) **Equation of beats :** If two waves of equal amplitudes 'a' and slightly different frequencies n_1 and n_2 travelling in a medium in the same direction are.

$$y_1 = a \sin \omega_1 t = a \sin 2\pi n_1 t \,;\; y_2 = a \sin \omega_2 t = a \sin 2\pi n_2 t$$

By the principle of super position : $\vec{y} = \vec{y}_1 + \vec{y}_2$

$y = A \sin \pi (n_1 + n_2) t$ where $A = 2a \cos \pi (n_1 - n_2) t$ = Amplitude of resultant wave.

(6) **Beat frequency :** $n = n_1 \sim n_2$.

(7) **Beat period :** $T = \dfrac{1}{\text{Beat frequency}} = \dfrac{1}{n_1 \sim n_2}$

16.18 Determination of Unknown Frequency

Let n_2 is the unknown frequency of tuning fork B, and this tuning fork B produce x beats per second with another tuning fork of known frequency n_1.

As number of beat/sec is equal to the difference in frequencies of two sources, therefore $n_2 = n_1 \pm x$

The positive/negative sign of x can be decided in the following two ways :

By loading	By filing
If B is loaded with wax so its frequency decreases	**If B is filed, its frequency increases**
If number of beats decreases $n_2 = n_1 + x$	If number of beats decreases $n_2 = n_1 - x$
If number of beats Increases $n_2 = n_1 - x$	If number of beats Increases $n_2 = n_1 + x$
If number of beats remains unchanged $n_2 = n_1 + x$	If number of beats remains unchanged $n_2 = n_1 - x$

If number of beats becomes zero $n_2 = n_1 + x$	If number of beats becomes zero $n_2 = n_1 - x$
If *A* is loaded with wax its frequency decreases	**If *A* is filed, its frequency increases**
If number of beats decreases $n_2 = n_1 - x$	If number of beats decreases $n_2 = n_1 + x$
If number of beats increases $n_2 = n_1 + x$	If number of beats Increases $n_2 = n_1 - x$
If number of beats remains unchanged $n_2 = n_1 - x$	If number of beats remains unchanged $n_2 = n_1 + x$
If number of beats becomes zero $n_2 = n_1 - x$	If no of beats becomes zero $n_2 = n_1 + x$

Sample problems based on Superposition of waves

***Problem* 23.** The stationary wave produced on a string is represented by the equation $y = 5\cos\left(\frac{\pi x}{3}\right)\sin(40\pi t)$ where x and y are in *cm* and t is in seconds. The distance between consecutive nodes is

(a) 5 *cm* (b) π *cm* (c) 3 *cm* (d) 40 *cm*

Solution : (c) By comparing with standard equation of stationary wave

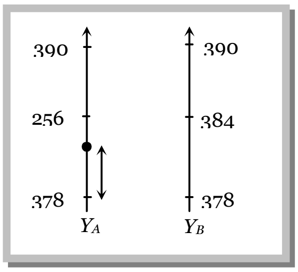
$$y = a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

We get $\frac{2\pi x}{\lambda} = \frac{\pi x}{3} \Rightarrow \lambda = 6$; Distance between two consecutive nodes = $\frac{\lambda}{2} = 3\ cm$

***Problem* 24.** On sounding tuning fork *A* with another tuning fork B of frequency 384 *Hz*, 6 beats are produced per second. After loading the prongs of *A* with wax and then sounding it again with *B*, 4 Beats are produced per second what is the frequency of the tuning fork *A*.

(a) 388 *Hz* (b) 80 *Hz* (c) 378 *Hz* (d) 390 *Hz*

Solution : (c)



Probable frequency of A is 390 *Hz* and 378 *Hz* and After loading the beats are decreasing from 6 to 4 so the original frequency of A will be $n_2 = n_1 - x = 378$ *Hz*.

***Problem* 25.** Two sound waves of slightly different frequencies propagating in the same direction produces beats due to **[MP PET 2000]**

(a) Interference (b) Diffraction (c) Polarization (d) Refraction

Solution : (a)

***Problem* 26.** Beats are produced with the help of two sound waves on amplitude 3 and 5 units. The ratio of maximum to minimum intensity in the beats is **[MP PMT 1999]**

(a) 2 : 1 (b) 5 : 3 (c) 4 : 1 (d) 16 : 1

Solution : (d) $\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{5+3}{5-3}\right)^2 = 16:1$

***Problem* 27.** Two tuning forks have frequencies 380 and 384 hertz respectively. When they are sounded together, they produce 4 beats. After hearing the maximum sound, how long will it take to hear the minimum sound **[MP PMT/PET 1998]**

(a) 1/2 sec (b) 1/4 *sec* (c) 1/8 *sec* (d) 1/16 *sec*

Solution : (c) Beats period = Time interval between two minima

$$T = \frac{1}{n_1 - n_2} = \frac{1}{4} \text{ sec}$$

Time interval between maximum sound and minimum sound = $T/2 = 1/8$ *sec*

***Problem* 28.** Two tuning fork A and B give 4 beats per second when sounded together. The frequency of A is 320 *Hz*. When some wax is added to B and it is sounded with A, 4 beats per second are again heard. The frequency of B is

(a) 312 *Hz* (b) 316 *Hz* (c) 324 *Hz* (d) 328 *Hz*

Solution : (c) Since there is no change in beats. Therefore the original frequency of B is

$n_2 = n_1 + x = 320 + 4 = 324$

***Problem* 29.** 41 forks are so arranged that each produces 5 beat/sec when sounded with its near fork. If the frequency of last fork is double the frequency of first fork, then the frequencies of the first and last fork respectively

[MP PMT 1997]

(a) 200, 400 (b) 205, 410 (c) 195, 390 (d) 100, 200

Solution : (a) Let the frequency of first tuning fork = n and that of last = $2n$

$n, n + 5, n + 10, n + 15$ ...... $2n$ this forms A.P.

Formula of A.P $l = a + (N-1)\,r$ where l = Last term, a = First term, N = Number of term, r = Common difference

$2n = n + (41 - 1)\,5$

$2n = n + 200$

$n = 200$ and $2n = 400$

***Problem* 30.** In stationary waves, antinodes are the points where there is **[MP PMT 1996]**

(a) Minimum displacement and minimum pressure change

(b) Minimum displacement and maximum pressure change

(c) Maximum displacement and maximum pressure change

(d) Maximum displacement and minimum pressure change

Solution : (d) At Antinodes displacement is maximum but pressure change is minimum.

***Problem* 31.** The equation $y = 0.15 \sin 5x \cos 300\, t$, describes a stationary wave. The wavelength of the stationary wave is

[MP PMT 1995]

(a) Zero *meter* (b) 1.256 *meter* (c) 2.512 *meter* (d) 0.628 *meter*

Solution : (b) By comparing with standard equation $\therefore \frac{2\pi x}{\lambda} = 5x \Rightarrow \lambda = \frac{2}{5} \times \pi = 1.256$ *meter*

***Problem* 32.** The equation of a stationary wave is $y = 0.8 \cos\left(\frac{\pi x}{20}\right) \sin 200\,\pi\, t$ where x is in *cm*. and t is in sec. The separation between consecutive nodes will be

(a) 20 *cm* (b) 10 *cm* (c) 40 *cm* (d) 30 *cm*

Solution : (a) Standard equation $y = A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda}$

By comparing this equation with given equation. $\frac{2\pi x}{\lambda} = \frac{\pi x}{20} \Rightarrow \lambda = 40$ *cm*

Distance Between two nodes = $\frac{\lambda}{2} = 20$ *cm*.

Problem 33. Which of the property makes difference between progressive and stationary waves

(a) Amplitude (b) Frequency (c) Propagation of energy (d) Phase of the wave

Solution : (c) In stationary waves there is no transfer of energy.

Problem 34. If amplitude of waves at distance r from a point source is A, the amplitude at a distance $2r$ will be **[MP PMT 1985]**

(a) $2A$ (b) A (c) $A/2$ (d) $A/4$

Solution : (c) $I \propto A^2$ and $I \propto \frac{1}{r^2}$ so $r \propto \frac{1}{A}$; $\frac{r_1}{r_2} = \frac{A_2}{A_1} \Rightarrow A_2 = A_1 \left(\frac{r_1}{r_2}\right) = A\left(\frac{1}{2}\right) = A/2$

Problem 35. If two waves of same frequency and same amplitude respectively on superimposition produced a resultant disturbance of the same amplitude the wave differ in phase by **[MP PMT 1990]**

(a) π (b) $2\pi/3$ (c) $\pi/2$ (d) zero

Solution : (b) $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$A^2 = A^2 + A^2 + 2A^2\cos\phi$ $[A_1 = A_2 = A$ given$]$

$\cos\phi = -1/2 \Rightarrow \phi = 120° = \frac{2\pi}{3}$

Problem 36. The superposition takes place between two waves of frequency f and amplitude a. The total intensity is directly proportional to **[MP PMT 1986]**

(a) a (b) $2a$ (c) $2a^2$ (d) $4a^2$

Solution : (d) $I \propto (a_1 + a_2)^2$ [As $a_1 = a_2 = a$]

$I \propto 4a^2$

Problem 37. The following equation represent progressive transverse waves **[MP PET 1993]**

$z_1 = A\cos(\omega t - kx)$

$z_2 = A\cos(\omega t + kx)$

$z_3 = A\cos(\omega t + ky)$

$z_4 = A\cos(2\omega t - 2ky)$

A stationary wave will be formed by superposing

(a) z_1 and z_2 (b) z_1 and z_4 (c) z_2 and z_3 (d) z_3 and z_4

Solution : (a) The direction of wave must be opposite and frequencies will be same then by superposition, standing wave formation takes place.

Problem 38. When two sound waves with a phase difference of $\pi/2$ and each having amplitude A and frequency ω are superimposed on each other, then the maximum amplitude and frequency of resultant wave is **[MP PMT 1989]**

(a) $\frac{A}{\sqrt{2}};\omega/2$ (b) $\frac{A}{\sqrt{2}};\omega$ (c) $\sqrt{2}A;\frac{\omega}{2}$ (d) $\sqrt{2}A;\omega$

Solution : (d) Resultant Amplitude = $\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} = \sqrt{A^2 + A^2 + 2A^2\cos\frac{\pi}{2}} = \sqrt{2}A$

and frequency remains same = ω.

Problem 39. There is a destructive interference between the two waves of wavelength λ coming from two different paths at a point. To get maximum sound or constructive interference at that point, the path of one wave is to be increased by **[MP PET 1985]**

(a) $\lambda/4$ (b) $\lambda/2$ (c) $\frac{3\lambda}{4}$ (d) λ

Solution : (b) Destructive interference means the path difference is $(2n-1)\frac{\lambda}{2}$

If it is increased by $\lambda/2$

Then new path difference $(2n-1)\frac{\lambda}{2}+\frac{\lambda}{2}=n\lambda$

which is the condition of constructive interference.

***Problem* 40.** The tuning fork and sonometer wire were sounded together and produce 4 beats/second when the length of sonometer wire is 95 *cm* or 100 *cm*. The frequency of tuning fork is **[MP PMT 1990]**

(a) 156 *Hz* (b) 152 *Hz* (c) 148 *Hz* (d) 160 *Hz*

Solution : (a) Frequency $n \propto \frac{1}{l}$ $\therefore$ As $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$

If n is the frequency of tuning fork.

$n+4 \propto \frac{1}{95} \Rightarrow n-4 \propto \frac{1}{100} \Rightarrow (n+4)\,95=(n-4)\,100 \Rightarrow n=156\ Hz.$

***Problem* 41.** A tuning fork F_1 has a frequency of 256 *Hz* and it is observed to produce 6 beats/second with another tuning fork F_2. When F_2 is loaded with wax. It still produces 6 beats/second with F_1. The frequency of F_2 before loading was **[MP PET 1990]**

(a) 253 *Hz* (b) 262 *Hz* (c) 250 *Hz* (d) 259 *Hz*

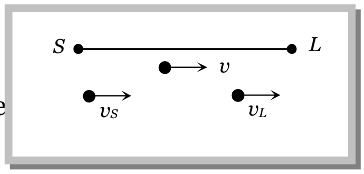
Solution : (b) No of beats does not change even after loading then $n_2=n_1+x=256+6=262\ Hz.$

16.19 Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

When the distance between the source and listener is decreasing the apparent frequency increases. It means the apparent frequency is more than the actual frequency of sound. The reverse is also true.

General expression for apparent frequency $n'=\frac{[(v+v_m)-v_L]n}{[(v+v_m)-v_S]}$



Here n = Actual frequency; v_L = Velocity of listener; v_S = Velocity of source

v_m = Velocity of medium and v = Velocity of sound wave

Sign convention : All velocities

along the direction S to L are taken as positive and all velocities along the direction L to S are taken as negative. If the medium is stationary $v_m=0$ then $n'=\left(\frac{v-v_L}{v-v_S}\right)n$

Special cases :

(1) Source is moving towards the listener, but the listener at rest $n'=\frac{v}{v-v_S}.n$

(2) Source is moving away from the listener but the listener is at rest $n'=\frac{v}{v+v_S}.n$

(3) Source is at rest and listener is moving away from the source $n' = \frac{v - v_L}{v} n$

(4) Source is at rest and listener is moving towards the source $n' = \frac{v + v_L}{v} .n$

(5) Source and listener are approaching each other $n' = \left(\frac{v + v_L}{v - v_S}\right) n$

(6) Source and listener moving away from each other $n' = \left(\frac{v - v_L}{v + v_S}\right) n$

(7) Both moves in the same direction with same velocity $n' = n$, *i.e.* there will be no Doppler effect because relative motion between source and listener is zero.

(8) Source and listener moves at right angle to the direction of wave propagation. $n' = n$

It means there is no change in frequency of sound heard if there is a small displacement of source and listener at right angle to the direction of wave propagation but for a large displacement the frequency decreases because the distance between source of sound and listener increases.

Important points

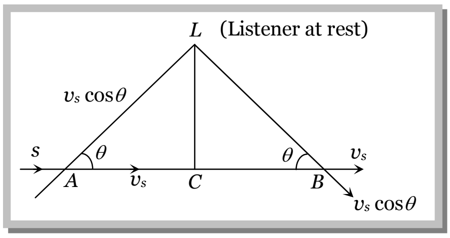
(i) If the velocity of source and listener is equal to or greater than the sound velocity then Doppler effect is not seen.

(ii) Doppler effect gives information regarding the change in frequency only. It does not says about intensity of sound.

(iii) Doppler effect in sound is asymmetric but in light it is symmetric.

16.20 Some Typical Features of Doppler's Effect in Sound

(1) **When a source is moving in a direction making an angle θ *w.r.t.* the listener :** The apparent frequency heard by listener L at rest



When source is at point A is $n' = \frac{nv}{v - v_s \cos\theta}$

As source moves along AB, value of θ increases, $\cos\theta$ decreases, n' goes on decreasing.

At point C, $\theta = 90^o$, $\cos\theta = \cos 90^o = 0$, $n' = n$.

At point B, the apparent frequency of sound becomes $n'' = \frac{nv}{v + v_s \cos\theta}$

(2) **When a source of sound approaches a high wall or a hill** with a constant velocity v_s, the reflected sound propagates in a direction opposite to that of direct sound. We can assume that the source and observer are approaching each other with same velocity *i.e.* $v_s = v_L$

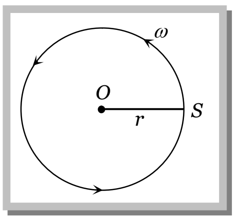
$$\therefore \quad n' = \left(\frac{v + v_L}{v - v_s}\right) n$$

(3) **When a listener moves between two distant sound sources :** Let v_L be the velocity of listener away from S_1 and towards S_2. Apparent frequency from S_1 is $n' = \frac{(v - v_L)n}{v}$

and apparent frequency heard from S_L is $n'' = \frac{(v + v_L)n}{v}$

$\therefore$ Beat frequency $= n'' - n' = \frac{2nv_L}{v}$

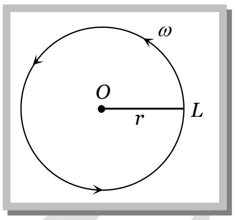
(4) **When source is revolving in a circle and listener L is on one side**



$v_s = r\omega$ so $n_{max} = \dfrac{nv}{v - v_s}$ and $n_{min} = \dfrac{nv}{v + v_s}$

(5) **When listener L is moving in a circle and the source is on one side**

$v_L = r\omega$ so $n_{max} = \dfrac{(v + v_L)n}{v}$ and $n_{min} = \dfrac{(v - v_L)n}{v}$



(6) There will be no change in frequency of sound heard, if the source is situated at the centre of the circle along which listener is moving.

(7) **Conditions for no Doppler effect :** (i) When source (S) and listener (L) both are at rest.

(ii) When medium alone is moving.

(iii) When S and L move in such a way that distance between S and L remains constant.

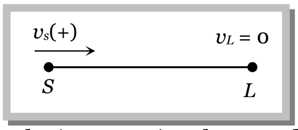
(iv) When source S and listener L, are moving in mutually perpendicular directions.

Sample problems based on Doppler effect

***Problem* 42.** A source of sound of frequency 90 *vibration/sec* is approaching a stationary observer with a speed equal to 1/10 the speed of sound. What will be the frequency heard by the observer **[MP PMT 2000]**

(a) 80 *vibration/sec* (b) 90 *vibration/sec* (c) 100 *vibration/sec* (d) 120 *vibration/sec*

Solution : (c) $n' = \dfrac{v}{v - v_s}.n \Rightarrow n' = \dfrac{v}{v - \dfrac{v}{10}}.n \Rightarrow n' = \dfrac{10}{9}n = \dfrac{10 \times 90}{9} = 100$ vibration/sec



***Problem* 43.** A source of sound of frequency 500 *Hz* is moving towards an observer with velocity 30 *m/s*. The speed of the sound is 330 *m/s*. The frequency heard by the observer will be **[MP PET 2000]**

(a) 550 *Hz* (b) 458.3 *Hz* (c) 530 *Hz* (d) 545.5 *Hz*

Solution : (a) $n' = \dfrac{v}{v - v_s}.n \Rightarrow n' = \dfrac{330}{330 - 30}.500 \Rightarrow n' = 550\ Hz$

***Problem* 44.** A motor car blowing a horn of frequency 124 *vibration/sec* moves with a velocity 72 *km/hr* towards a tall wall. The frequency of the reflected sound heard by the driver will be (velocity of sound in air is 330 *m/s*) **[MP PET 1997]**

(a) 109 *vibration/sec* (b) 132 *vibration/sec* (c) 140 *vibration/sec* (d) 248 *vibration/sec*

Solution : (c) In the given condition source and listener are at the same position *i.e.* (car) for given condition

$n' = \dfrac{v + v_{car}}{v - v_{car}}.n = \dfrac{330 + 20}{330 - 20}.n = 140$ *vibration/sec*

***Problem* 45.** The driver of car travelling with a speed 30 *meter/sec.* towards a hill sounds a horn of frequency 600 *Hz*. If the velocity of sound in air is 330 m/s the frequency of reflected sound as heard by the driver is **[MP PMT 1996]**

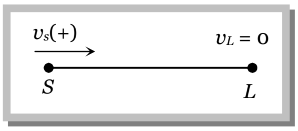
(a) 720 *Hz* (b) 555.5 *Hz* (c) 550 *Hz* (d) 500 *Hz*

Solution : (a) This question is same as that of previous one so $n' = \dfrac{v + v_{car}}{v - v_{car}}.n = 720\ Hz$

***Problem* 46.** The source of sound s is moving with a velocity 50 *m/s* towards a stationary observer. The observer measures the frequency of the source as 1000 *Hz*. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ? The velocity of sound in the medium is 350 *m/s* **[MP PMT**

(a) 750 *Hz* (b) 857 *Hz* (c) 1143 *Hz* (d) 1333 *Hz*

Solution : (a) When source is moving towards the stationary listener.



$n' = \frac{v}{v - v_s} n \Rightarrow 1000 = \frac{350}{350 - 50}.n \Rightarrow n = 857.14$

When source is moving away from the stationary observer $n'' = \frac{v}{v + v_s} = \frac{350}{350 + 50} \times 857 = 750\ Hz$

Problem **47.** A source and listener are both moving towards each other with speed $v/10$ where v is the speed of sound. If the frequency of the note emitted by the source is f, the frequency heard by the listener would be nearly **[MP PMT 1994]**

(a) $1.11f$ (b) $1.22f$ (c) f (d) $1.27f$

Solution : (b) $n' = \left(\frac{v + v_L}{v - v_s}\right) n \Rightarrow n' = \left(\frac{v + \frac{v}{10}}{v - \frac{v}{10}}\right) n \Rightarrow n' = \frac{11}{9} f = 1.22f.$

Problem **48.** A man is watching two trains, one leaving and the other coming in with equal speed of 4 m/s. If they sound their whistles, each of frequency 240 Hz, the number of beats heard by the man (velocity of sound in air = 320 m/s) will be equal to **[MP PET 1999; CPMT 1997; NCERT 1984]**

(a) 6 (b) 3 (c) 0 (d) 12

Solution : (a) App. Frequency due to train which is coming in $n_1 = \frac{v}{v - v_s}.n$

App. Frequency due to train which is leaving $n_2 = \frac{v}{v + v_s}.n$

So number of beats $n_1 - n_2 = \left(\frac{1}{316} - \frac{1}{324}\right) 320 \times 240 \Rightarrow n_1 - n_2 = 6$

Problem **49.** At what speed should a source of sound move so that observer finds the apparent frequency equal to half of the original frequency **[RPMT 1996]**

(a) $v/2$ (b) $2v$ (c) $v/4$ (d) v

Solution : (d) $n' = \frac{v}{v + v_s}.n \Rightarrow \frac{n}{2} = \frac{v}{v + v_s}.n \Rightarrow v_s = v$